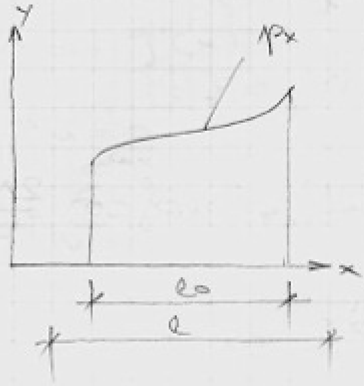
4.10.2005.

За ослобађање од писменог потребна је 75% на сваком колоквијуму.

Развијање ф-ја у тригонометријске редове:



Претпоставимо да $p(x)$ можемо написати у ф-ји бесконачног реда:

a_i = непознати коеф. које треба одредити

φ_i = познате ф-је (тригонометријске – триг. редови)

$$p(x) = \sum_{i=0}^{\infty} a_i \varphi_i(x)$$

$a_i = ?$ (одређује се помоћу методе најмањих квадрата)

min R = грешка

$$\min R(a_i) = \min \int_l (p_x - p(x))^2 \, dx \qquad \frac{\partial R(a_i)}{\partial a_i} = 0$$

$$\int_l (p_x - p(x)) \cdot \frac{\partial p(x)}{\partial a_i} \, dx = 0 \, , \qquad i = 0,1,2,3\ldots$$

проблем се поједностављује ако су ф-је $\varphi_i(x)$ ортогоналне тј.:

$$\int_l \varphi_n(x) \varphi_m(x) \, dx = 0 \qquad m \neq n$$
$$= 1 \qquad m = n$$

sin(x) ∧ cos(x) – тригонометријске ф-је:

$p(x) = p(x+l)$

$$p(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2n\pi}{l} \cdot x + b_n \sin \frac{2n\pi}{l} \cdot x \right)$$

Општи период L може се изабрати произво-
љно у зависности од проблема:

$L = l, 2l, 3l, 4l \ldots$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2n\pi}{l} \cdot x + b_n \sin \frac{2n\pi}{l} \cdot x\right) \qquad (*)$$

a_0 - добијамо интегралећи овај израз:

$$\int_x^{x+L} f(x)\,dx = \frac{a_0}{2} \int_x^{x+L} dx + a_1 \int_x \cos\left(\frac{2\pi}{L} x\right) dx + \ldots + a_n \int_x^{x+L} \cos \frac{2n\pi}{L} x\,dx$$

$$+ \ldots + b_1 \int_x^{x+L} \sin \frac{2\pi}{L} x\,dx + b_n \int_x^{x+L} \sin \frac{2n\pi}{L} x\,dx$$

$$\int_x^{x+L} f(x)\,dx = \frac{a_0}{2} \cdot L \qquad \boxed{a_0 = \frac{2}{L} \int_x^{x+L} f(x)\,dx}$$

Да бисмо добили коефицијенте a_n, израз (*) ће-
мо помножити са $\cos \frac{2n\pi}{L} \cdot x$ и интегралимо:

$$\int_x^{x+L} f(x) \cdot \cos \frac{2n\pi}{L} x\,dx = \frac{a_0}{2} \overset{\to 0}{\int_x^{x+L} \cos \frac{2n\pi}{L}\,dx} + a_n \int_x^{x+L} \cos^2 \frac{2n\pi}{L} x\,dx$$

$$\boxed{a_n = \frac{2}{L} \int_x^{x+L} f(x) \cos \frac{2n\pi}{L} x\,dx}$$

$$\int_x^{x+L} \cos \frac{2n\pi}{L} x \cos \frac{2m\pi x}{L}\,dx = \begin{cases} 0 & m \neq n \\ \frac{L}{2} & m = n \end{cases}$$

$$\int_x^{x+L} \cos \frac{2n\pi}{L} x \cdot \sin \frac{2m\pi}{L} x\,dx = 0 \qquad \forall\, m, n$$

$$\int_x^{x+L} \sin \frac{2n\pi}{L} \cdot x \cdot \sin \frac{2m\pi}{L} x\,dx = \begin{cases} 0, & m \neq n \\ \frac{L}{2}, & m = n \end{cases}$$

$$\boxed{b_n = \frac{2}{L} \int_x^{x+L} f(x) \sin \frac{2n\pi}{L} \cdot x\,dx}$$

ф-ја је парна ако је симетрична у односу на
y-осу

$\boxed{f(x) = f(-x)}$ - парна ф-ја. ✓

$b_n = 0$ $\quad \boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{2n\pi}{L} x}$ ✓

$$\boxed{a_n = \frac{4}{L} \int_0^{L/2} f(x) \cdot \cos \frac{2n\pi}{L} \cdot x \, dx} \quad V$$

$f(-x) = -f(x)$ непарна ф-ја ✓

$a_0 = a_n = 0$

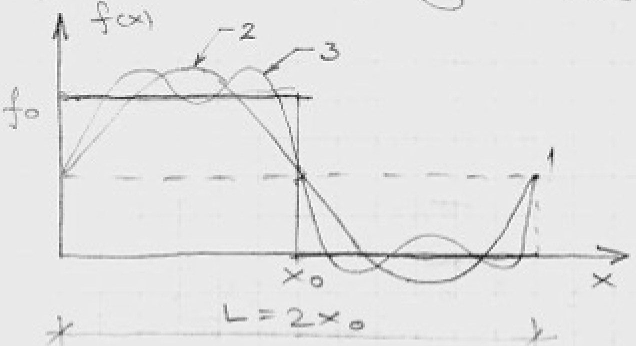
$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{2n\pi}{L} x \qquad b_n = \frac{4}{L} \int_0^{L/2} f(x) \sin \frac{2n\pi}{L} x \, dx$$

1. Дату ф-ју развити у тригоном ред на 3 начина:

a) тако да садржи синусне и косинусне ф-је;

b) као непарну ф-ју;

c) као парну ф-ју.



$: y = f_0$

$0 < x < x_0$

$x > x_0 \quad y = 1$ усвојени период

a) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{2n\pi}{L} x + \sum_{n=1}^{\infty} b_n \sin \frac{2n\pi}{L} x \qquad L = 2x_0$

$$a_0 = \frac{2}{L} \int_0^L f(x) \cdot dx = \frac{2}{2x_0} \int_0^{2x_0} f(x) \, dx = \frac{1}{x_0} \int_0^{x_0} f_0 \, dx = \frac{f_0}{x_0} x \Big|_0^{x_0} = f_0$$

$$a_n = \frac{2}{2x_0} \int_0^{2x_0} f(x) \cos \frac{2n\pi}{2x_0} x \, dx = \frac{1}{x_0} \int_0^{x_0} f_0 \cos \frac{n\pi}{x_0} x \, dx =$$

$$= \frac{f_0}{x_0} \frac{x_0}{n\pi} \cdot \sin \frac{n\pi x}{x_0} \Big|_0^{x_0} = \frac{f_0}{n\pi} \cdot (\overset{0, \forall n}{\sin n\pi} - 0) = 0$$

$$b_n = \frac{2}{2x_0} \int_0^{2x_0} f(x) \sin \frac{2n\pi}{2x_0} x \, dx = \frac{1}{x_0} \int_0^{x_0} f_0 \sin \frac{n\pi}{x_0} x \, dx =$$

$$= \frac{f_0}{x_0} \left(-\frac{x_0}{n\pi}\right) \cos \frac{n\pi}{x_0} x \Big|_0^{x_0} = -\frac{f_0}{n\pi} \cdot (\cos n\pi - 1) = \begin{cases} \frac{2f_0}{n\pi}; & n = 1, 3, \ldots \\ 0; & n = 0, 2, \ldots \end{cases}$$

$$f(x) \approx \frac{f_0}{2} + \frac{2f_0}{\pi} \left(\sin \frac{\pi x}{x_0} + \frac{1}{3} \sin \frac{3\pi x}{x_0} + \ldots\right.$$

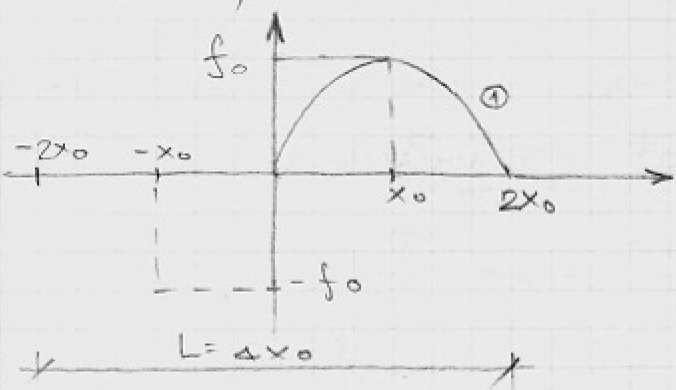
Сада провјеравамо колико добро смо апроксимирали ф-ју, ако узмемо само један члан реда, два, три

$f(x) \approx \frac{f_0}{2}$ - осредњена ф-ја

$f(x) \approx \frac{f_0}{2} + \frac{2f_0}{\pi} \cdot \sin \frac{\pi x}{x_0}$

b) $f(-x) = -f(x)$

Нама ф-ја није ни парна ни непарна па треба да је допунимо да би била непарна (парна)



Непарна је симетрична у односу на коорд. почетак

$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{2n\pi}{L} x \qquad b_n = \frac{4}{L} \int_0^{L/2} f(x) \sin \frac{2n\pi}{L} x \, dx$

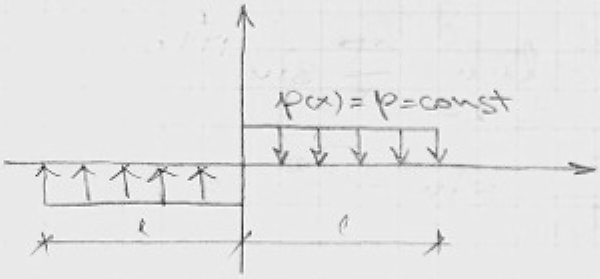
$b_n = \frac{4}{4x_0} \int_0^{2x_0} f(x) \sin \frac{2n\pi}{4x_0} x \, dx =$

$= \frac{1}{x_0} \int_0^{x_0} f_0 \sin \frac{n\pi}{2x_0} x \, dx = \frac{f_0}{x_0} \left(-\frac{2x_0}{n\pi}\right) \cdot \cos \frac{n\pi}{2x_0} x \Big|_0^{x_0} =$

$$= -\frac{2f_0}{n\pi}\left(\cos \frac{n\pi}{2} - 1\right) = \begin{cases} \frac{2f_0}{n\pi}, & n = 1,3,5\ldots \\ \frac{4f_0}{n\pi}, & n = 2,6,10\ldots \\ 0, & n = 4,8,12 \end{cases}$$

$$f(x) = \frac{2f_0}{\pi}\left(\sin \frac{\pi}{2x_0} + \sin \frac{\pi}{x_0} x + \frac{1}{3} \sin \frac{3\pi}{2x_0} x + \frac{1}{5} \sin \frac{5\pi x}{2x_0} + \ldots\right.$$

2. Развити као непарну ф-ју (линијско оптерећење)



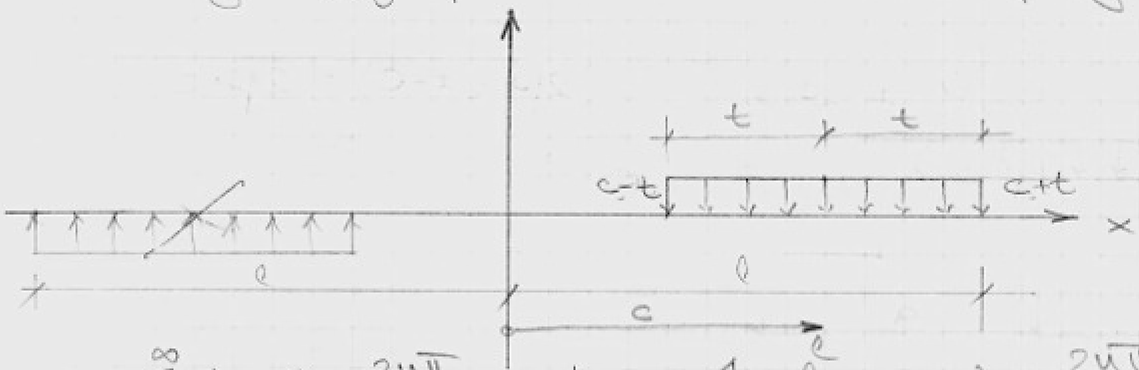
$$p(-x) = -p(x)$$

$$p(x) = \sum_{n=1}^{\infty} b_n \sin \frac{2n\pi}{L} x$$

$$b_n = \frac{4}{2l} \int_0^l p \sin \frac{2n\pi}{2l} x \, dx = \frac{2p}{l} \left(-\frac{l}{n\pi}\right) \cdot \cos \frac{n\pi}{l} x \Big|_0^l =$$

$$= -\frac{2p}{n\pi} (\cos n\pi - 1) = \begin{cases} \frac{4p}{n\pi} & ; n = 1, 3, 5 \ldots \\ 0 & ; n = 2, 4, 6 \ldots \end{cases}$$

3. Дату ф-ју развити као непарну:



$$p(x) = \sum_{n=1}^{\infty} b_n \sin \frac{2n\pi}{L} x \qquad b_n = \frac{4}{2l} \int_0^l p(x) \sin \frac{2n\pi}{2l} x \, dx =$$

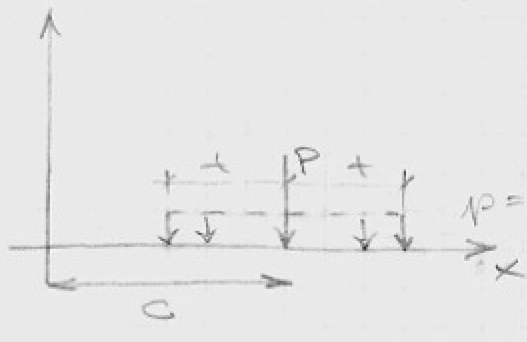
$$= \frac{2}{l} p \int_{c-t}^{c+t} \sin \frac{n\pi}{l} \cdot x \, dx = \frac{2p}{l} \cdot \left(-\frac{l}{n\pi}\right) \cdot \cos \frac{n\pi}{l} x \Big|_{c-t}^{c+t} =$$

$$= -\frac{2p}{n\pi} \left(\cos \frac{n\pi(c+t)}{l} - \cos \frac{n\pi(c-t)}{l}\right).$$

~~непарну~~

Домаћи: исту ову ф-ју развити као парну:

4. Дату ф-ју развити у триг. ред



$$p = \frac{P}{2t}$$

ф-ја има вриједност само у једној тачки

Развлачимо ф-ју на неки интервал 2t

$$\lim_{t\to 0} b_n = \frac{4p}{n\pi}\sin\frac{n\pi c}{l}\sin\frac{n\pi t}{l} = \lim_{t\to 0}\frac{4p}{n\pi}\sin\frac{n\pi c}{l}\cdot\frac{\sin\frac{n\pi t}{l}}{\frac{n\pi t}{l}}\cdot\frac{n\pi t}{l} = \lim_{t\to 0}\frac{4p}{n\pi}\cdot\frac{n\pi t}{l}\cdot\sin\frac{n\pi c}{l} =$$

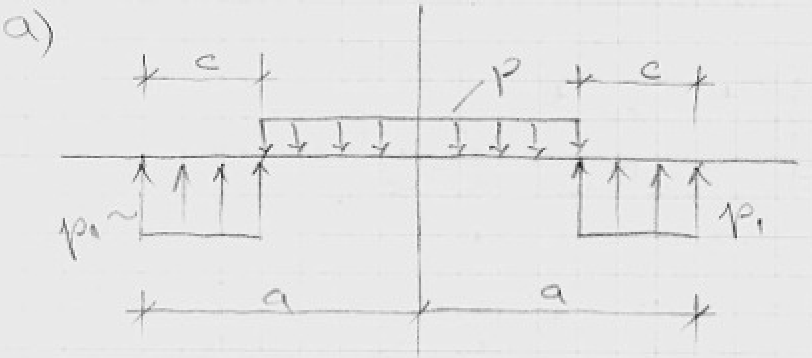
$$= \frac{2p}{l}\sin\frac{n\pi c}{l} \qquad p(x) = \sum_{n=1}^{\infty}\frac{2p}{l}\sin\frac{n\pi c}{l}\sin\frac{n\pi x}{l}$$

5. Дату ф-ју оптерећења развити у триг. ред као парну ф-ју ако је:

а) период $L=2a$

б) период $L=4a$ (домаћи)

а)



$$2p(a-c) = 2p_1 c$$

$$p_1 = p\,\frac{a-c}{c}$$

Ово је ивично оптерећење које се јавља код зидних носача

$L=2a$

$$p(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2n\pi}{2a}x$$

$a_0=0$ зато што имамо случај равнотежног оптерећења

$$a_n = \frac{4}{2a}\int_0^a p(x)\cos\frac{2n\pi}{2a}x\,dx =$$

$$= \frac{2}{a}\left(\int_0^{a-c} p\cdot\cos\frac{n\pi}{a}x\,dx - \int_{a-c}^{a} p\,\frac{a-c}{c}\cos\frac{n\pi x}{a}\,dx\right) = \dots$$

$$= \frac{2p\cdot a}{n\pi c}\sin\frac{n\pi c}{a}\cos n\pi = (-1)^n$$

Домаћи:

1)
$$f(x) = \begin{cases} f_0 , & 0 \le x \le x_0 \\ 0 , & x_0 < x \le 2x_0 \end{cases}$$

развити као парну ф-ју!

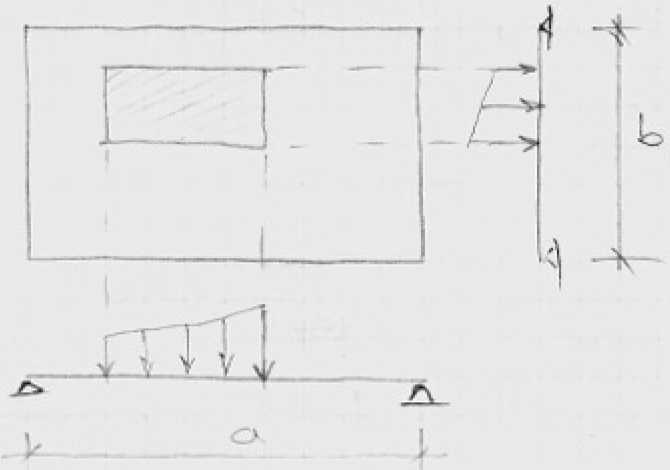
11.10.2005.

Развијање ф-је двије промјенљиве у тригонометријски ред:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2n\pi}{L} x + b_n \sin \frac{2n\pi}{L} x \right)$$

Површинско оптерећење је ф-ја двије промјенљиве

$p(x,y)$



$$p(x,y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left(A_{mn} \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} y + B_{mn} \cos \frac{m\pi}{a} x \cdot \cos \frac{n\pi}{b} y \right)$$

$L_x = 2a$

$L_y = 2b$

$$\boxed{p(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} y} \Big/ \cdot \sin \frac{r\pi x}{a} \sin \frac{s\pi y}{b} \Big/ \int_0^a \int_0^b$$

$$\int_0^a \int_0^b p(x,y) \sin \frac{r\pi x}{a} / \sin \frac{s\pi}{b} y \, dx\, dy =$$

$$= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \int_0^a \int_0^b \sin \frac{m\pi}{a} x \sin \frac{r\pi}{a} x \cdot \sin \frac{n\pi}{b} y \cdot \sin \frac{s\pi}{b} y \, dx\, dy$$

користимо услове ортогоналности:

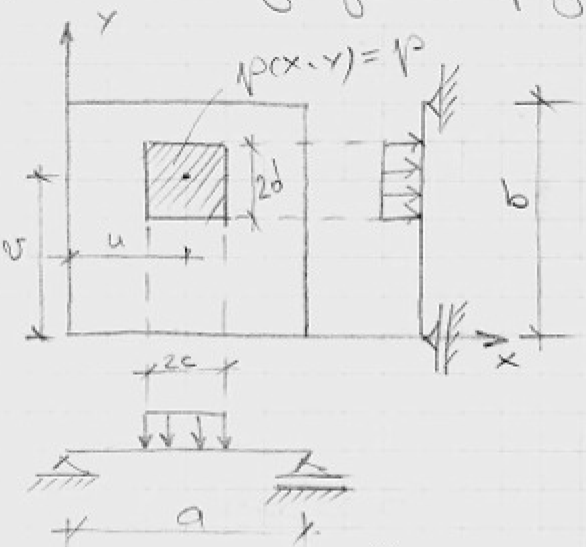
$$\int_0^a \sin\frac{m\pi x}{a}\sin\frac{r\pi}{a}x\,dx = \begin{cases}\frac{a}{2}; & m=r\\ 0; & m\neq r\end{cases}$$

$$\int_0^b \sin\frac{n\pi}{b}y\sin\frac{s\pi}{b}y\,dy = \begin{cases}\frac{b}{2}; & n=s\\ 0; & n\neq s\end{cases}$$

$$\int_0^a\int_0^b p(x,y)\cdot\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\,dx\,dy = A_{mn}\frac{a}{2}\frac{b}{2}$$

$$\boxed{A_{mn} = \frac{4}{ab}\int_0^a\int_0^b p(x,y)\sin\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y\,dx\,dy}$$

1. Равномјерно расподјељено оптерећење унутар правоугаоне плоче димензија $2c\times 2d$ развити у двоструки тригонометријски ред



$$p(x,y) = \begin{cases} p; & u-c\le x\le u+c \quad v-d\le y\le v+d\\ 0; & \end{cases}$$

$$p(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y$$

$$A_{mn} = \frac{4}{ab}\int_0^a\int_0^b p(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dx\,dy =$$

$$= \frac{4}{ab}\int_{u-c}^{u+c}\int_{v-d}^{v+d} p\cdot\sin\frac{m\pi}{a}x\sin\frac{n\pi}{b}y\,dx\,dy =$$

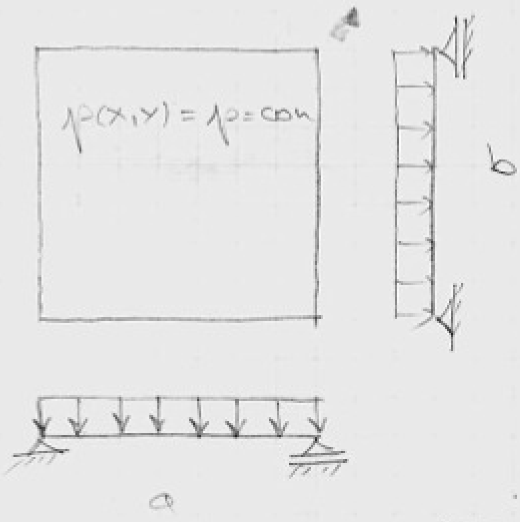
$$= \frac{4P}{ab} \int_{u-c}^{u+c} \sin \frac{m\pi}{a} x \, dx \int_{v-d}^{v+d} \sin \frac{n\pi}{b} y \, dy =$$

$$= \frac{4P}{ab} \left(-\frac{a}{m\pi}\right) \cdot \cos \frac{m\pi x}{a} \Big|_{u-c}^{u+c} \left(-\frac{b}{n\pi}\right) \cdot \cos \frac{n\pi}{b} y \Big|_{v-d}^{v+d}$$

$$A_{mn} = \frac{16P}{\pi^2 mn} \sin \frac{m\pi u}{a} \cdot \sin \frac{m\pi c}{a} \cdot \sin \frac{n\pi v}{b} \sin \frac{n\pi d}{b}$$

$\forall \; m, n = 1, 2, 3 \ldots$

2. Плоча равномјерно оптерећена по цијелој површини



$2c = a \qquad c = \frac{a}{2}$

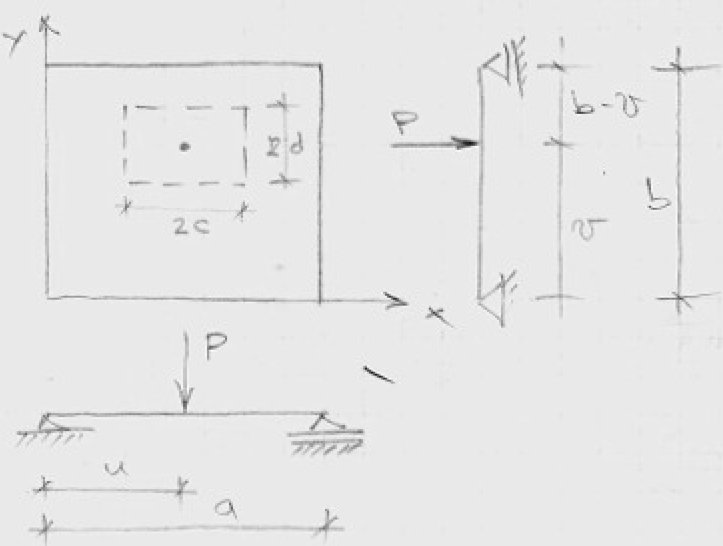
$2d = b \qquad d = \frac{b}{2}$

$u = \frac{a}{2} \qquad v = \frac{b}{2}$

$$A_{mn} = \frac{16p}{\pi^2 mn} \sin^2 \frac{m\pi}{2} \sin^2 \frac{n\pi}{2} = \begin{cases} \frac{16p}{\pi^2 mn}; & m, n = 1, 3, 5 \\ 0; & m, n = 2, 4, 6 \end{cases}$$

$$p(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{16p}{\pi^2 mn} \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y \quad ; \quad m = 1, 3, 5 \ldots n$$

3.



$$p = \frac{P}{4cd}$$

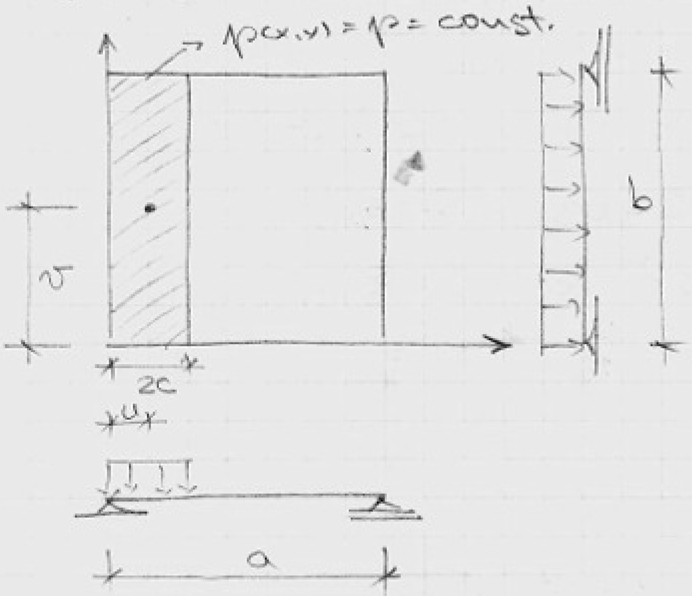
$$\lim_{\substack{c\to 0\\ d\to 0}} A_{mn} = \frac{16P}{\pi^2 mn}\sin\frac{m\pi u}{a}\sin\frac{m\pi c}{a}\cdot\sin\frac{n\pi v}{b}\cdot\sin\frac{n\pi d}{b}$$

$$= \lim_{\substack{c\to 0\\ d\to 0}} \frac{16p}{\pi^2 mn}\sin\frac{m\pi u}{a}\cdot\frac{\sin\frac{m\pi}{a}c}{\frac{m\pi c}{a}}\cdot\frac{m\pi c}{a}\cdot\sin\frac{n\pi v}{b}\cdot$$

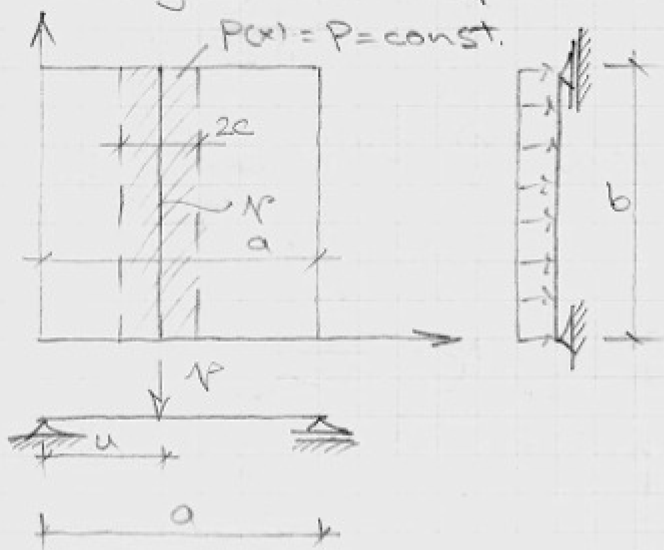
$$\cdot\frac{\sin\frac{n\pi d}{b}}{\frac{n\pi d}{b}}\cdot\frac{m\pi d}{b}$$

$$A_{mn} = \frac{4P}{ab}\sin\frac{m\pi}{a}u\cdot\sin\frac{n\pi}{b}v$$

4. Ломати:



5. линијски оптерећена плоча:



$$p = \frac{P}{2c}$$

$$\lim_{c\to 0} A_{mn} = \frac{16P}{\pi^2 mn}\sin\frac{m\pi}{a}u\,\sin\frac{m\pi}{a}c\,\sin^2\frac{n\pi}{2}$$

$$= \lim_{c\to 0}\frac{16P}{\pi^2 mn}\cdot\sin\frac{m\pi u}{a}\cdot\frac{\sin\frac{m\pi}{a}c}{\frac{m\pi c}{a}}\cdot$$

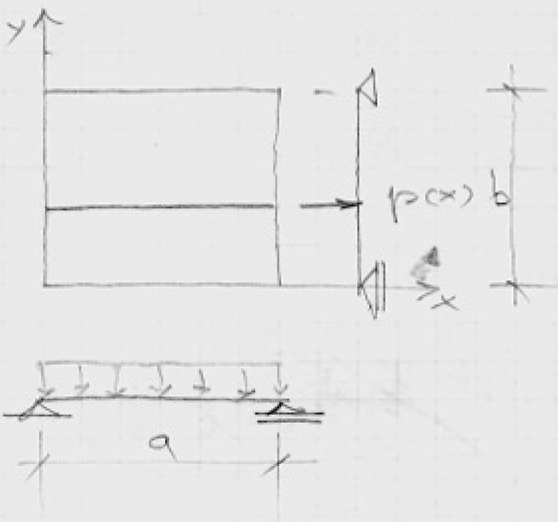
$$\cdot \frac{m\pi c}{a} \sin^2 \frac{n\pi}{2}$$

$$A_{mn} = \frac{8p}{\pi n a} \sin \frac{m\pi}{a} u \sin^2 \frac{n\pi}{2} = \begin{cases} \frac{8p}{an\pi} \sin \frac{m\pi}{a} u ; & n = 1,3,5 \\ 0 ; & n = 2,4,6 \ldots \end{cases}$$

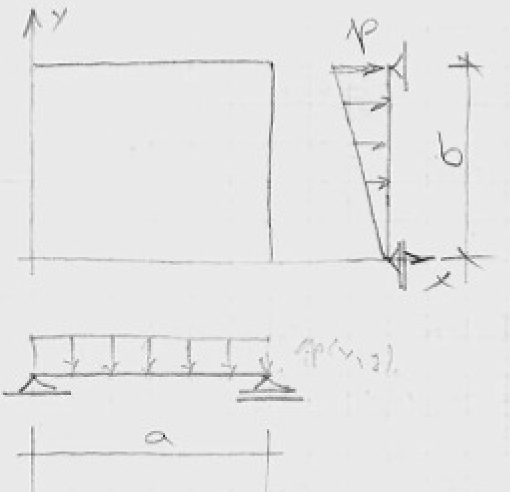
$$p(x) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{8p}{an\pi} \sin \frac{m\pi u}{a} \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} x$$

$m = 1,2,3 \ldots \qquad n = 1,3,5 \ldots$

6. Задатак



7.



Оптерећење се не мијења дуж x-осе, а мијења се линеарно дуж y-осе

$p(x,y) = p \cdot y/b$

$$p(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} y$$

$$A_{mn} = \frac{4}{ab} \int_0^a \int_0^b \frac{p}{b} y \cdot \sin \frac{m\pi}{a} x \sin \frac{n\pi}{b} y \, dx dy$$

$$= \frac{4p}{ab^2} \underbrace{\int_0^a \sin \frac{m\pi}{a} x \, dx}_{I_1} \underbrace{\int_0^b y \sin \frac{n\pi}{b} y \, dy}_{I_2}$$

$$I_1 = -\frac{a}{m\pi} \cos \frac{n\pi}{a} x \Big|_0^a = -\frac{a}{m\pi} (\cos m\pi - 1) = \begin{cases} \frac{2a}{m\pi} , & m = 1,3,5 \\ 0 ; & m = 2,4,6 \end{cases}$$

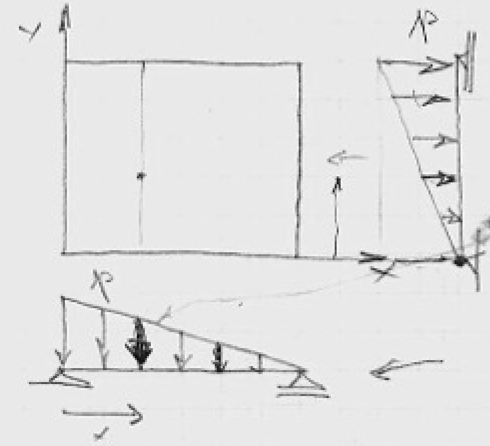
$$I_2 = \int_0^b y \sin\frac{n\pi}{b}y\,dy = \left| \begin{matrix} u = y & dv = \sin\frac{n\pi}{b}y\,dy \\ du = dy & v = -\frac{b}{n\pi}\cos\frac{n\pi}{b}y \end{matrix} \right|$$

$$I_2 = -\frac{b}{n\pi}\, y\cos\frac{n\pi}{b}y \Big|_0^b + \frac{b}{n\pi}\int_0^b \cos\frac{n\pi}{b}y\,dy =$$

$$= -\frac{b^2}{n\pi}(-1)^n \qquad A_{mn} = \begin{cases} \frac{4p}{ab}\cdot\frac{2a}{m\pi}\cdot\frac{b^2}{n\pi}(-1)^n, & m = 1,3,5\ldots = \frac{-8p}{\pi^2 mn}(-1)^n \\ 0, & m = 2,4,6\ldots \end{cases}$$

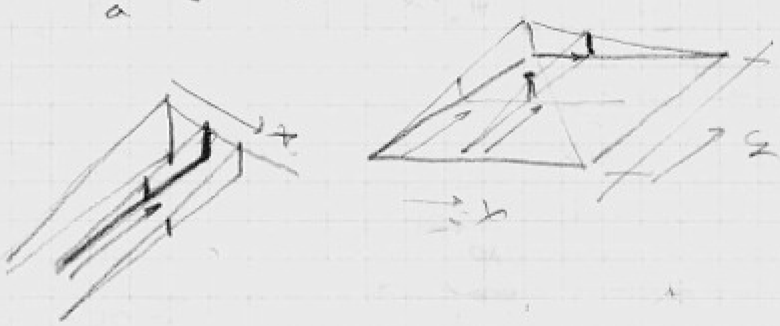
8.

Домашни:

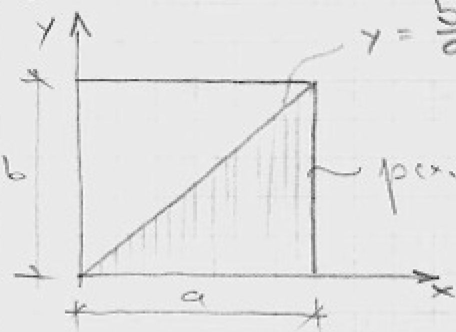


$$p(x,y) = \frac{p}{a}(a-x)\,\frac{y}{b}$$

$$\frac{p}{a}\cdot(a-x).$$



9.



$$y = \frac{b}{a}x$$

$$p(x,y) = p = \text{const.}$$

$$A_{mn} = \frac{4}{ab}\int_0^a\int_0^b p(x,y)\sin\frac{m\pi}{a}x\,\sin\frac{n\pi}{b}y\,dx\,dy =$$

$$= \frac{4}{ab}\int_0^a\int_0^{\frac{b}{a}x} p\cdot\sin\frac{m\pi}{a}x\cdot\sin\frac{n\pi}{b}y\,dx\,dy$$

(1) $\int_0^{\frac{b}{a}x}\sin\frac{n\pi}{b}y\,dy = \ldots = f(x)$

(2) $\int_0^a f(x)\sin\frac{m\pi}{a}x\,dx$

18.10.2005.

СИЛЕ У ПРЕСЈЕЦИМА ПЛОЧЕ:

$$\overline{N_x} = \int_F \sigma_x \, dy \, dz = \int_{-h/2}^{h/2} \sigma_x \, dy \, dz \; /: dy \qquad [kN]$$

$$N_x = \int_{-h/2}^{h/2} \sigma_x \, dz \quad [kN/m]$$

$$\overline{M_x} = \int_F \sigma_x \, dy \, dz \cdot z \quad [kNm] \quad /: dy$$

$$M_x = \int_{-h/2}^{h/2} \sigma_x \, dz \quad [kNm/m]$$

$$\overline{N_{xy}} = \int_F \tau_{xy} \, dy \, dz \quad [kN] \quad /: dy$$

$$N_{xy} = \int_{-h/2}^{h/2} \tau_{xy} \, dz \quad [kN/m]$$

$$\overline{M_{xy}} = \int_F \tau_{xy} \, dy \, dz \cdot z \quad [kNm] \quad /: dy$$

$$M_{xy} = \int_{-h/2}^{h/2} \tau_{xy} \, dz \cdot z \quad [kNm/m]$$

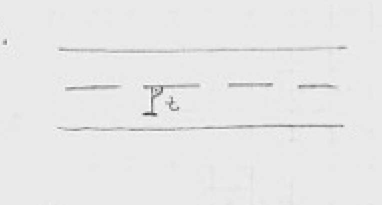
$$\overline{T_x} = \int_F \tau_{xz} \, dy \, dz \quad [kN] \quad /: dy$$

$$T_x = \int_{-h/2}^{h/2} \tau_{xz} \, dz \quad [kN/m]$$

Основне претпоставке теорије савијања плоча

Кирхофове претпоставке: 1., 2., 3. · имамо на папирима

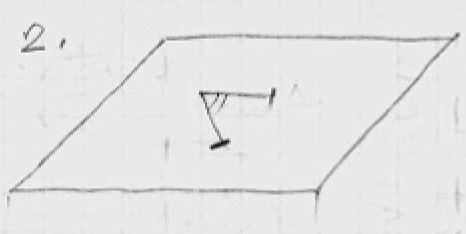
1.



$\gamma_{xz} = 0$

$\gamma_{yz} = 0$

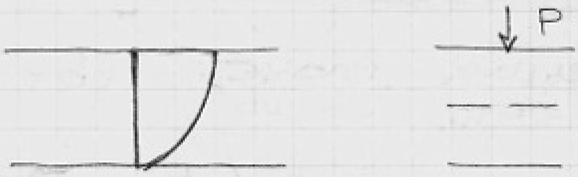
2.



$\varepsilon_z = 0$

$\frac{\partial w}{\partial z} = 0 \Rightarrow w = w(x, y)$

3. $\sigma_z \approx 0$



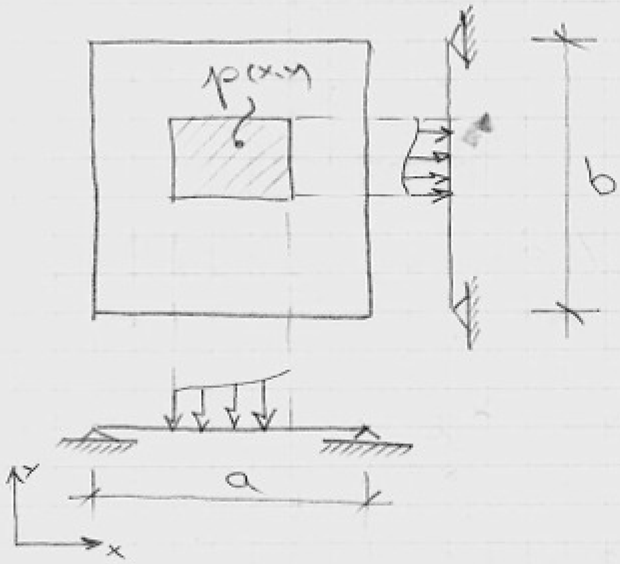
ОВАЈЕ СЕ
НЕ ЗАНЕМАРУЈЕ

$$K = \frac{E h^3}{12(1-\nu^2)}$$

ДИФ. ЈЕДН. САВИЈАЊА:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{z(x,y)}{K}$$

NAVIER-ово решење



$x=0$

$x=a$

$$\left\{ \begin{array}{l} w=0 \Rightarrow \frac{\partial w}{\partial y} \equiv 0 \quad \frac{\partial^2 w}{\partial y^2} = 0 \\ M_x = 0 \\ -K\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = 0 \end{array} \right.$$

$$\boxed{\frac{\partial^2 w}{\partial x^2} = 0}$$

НА ИСТИ НАЧИН

$$\boxed{\frac{\partial^2 w}{\partial y^2} = 0}$$

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \cdot \sin\frac{m\pi x}{a} \cdot \sin\frac{n\pi y}{b}$$

$$\frac{\partial^2 w}{\partial x^2} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \left(\frac{m\pi}{a}\right)^2 \cdot \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} = 0 \quad \text{ЗА } x=0$$

$$\frac{\partial^2 w}{\partial y^2} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \left(\frac{n\pi}{b}\right)^2 \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} = 0 \quad \text{ЗА } y=0$$

$$\frac{\partial^4 w}{\partial x^4} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \left(\frac{m\pi}{a}\right)^4 \sin\frac{m\pi}{a}x \sin\frac{n\pi}{b}y$$

$$\frac{\partial^4 w}{\partial y^4} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \left(\frac{n\pi}{b}\right)^4 \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$\frac{\partial^4 w}{\partial x^2 \partial y^2} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \left(\frac{m\pi}{a}\right)^2 \left(\frac{n\pi}{b}\right)^2 \sin\frac{m\pi}{a}x \sin\frac{n\pi y}{b}$$

$$z(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} z_{mn}\sin\frac{m\pi}{a}x\,\sin\frac{n\pi}{b}y$$

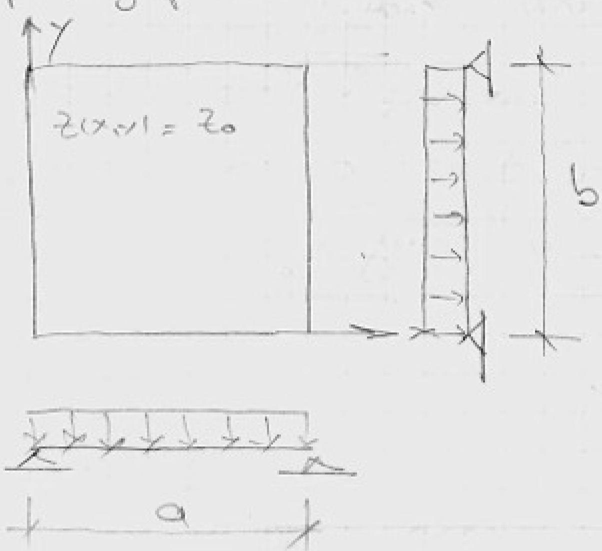
$$\boxed{z_{mn}=\frac{4}{ab}\int_0^a\int_0^b z(x,y)\sin\frac{m\pi}{a}x\,\sin\frac{n\pi}{b}y\,dx\,dy}$$

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\left[\left(\frac{m\pi}{a}\right)^4+2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2+\left(\frac{n\pi}{b}\right)^4\right]\cdot\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}=$$

$$=\frac{1}{k}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} z_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$A_{mn}\left(\frac{m^2\pi^2}{a^2}+\frac{n^2\pi^2}{b^2}\right)^2=\frac{z_{mn}}{k}\Rightarrow\boxed{A_{mn}=\frac{z_{mn}}{k\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}}$$

Примјер 1:



$$z_{mn}=\frac{16\,z_0}{\pi^2 mn}\qquad m,n=1,3,5,\dots$$

$$A_{mn}=\frac{16\,z_0}{k\pi^6\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2\cdot mn}$$

$$w(x,y)=\frac{16z_0}{k\pi^6}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{mn\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

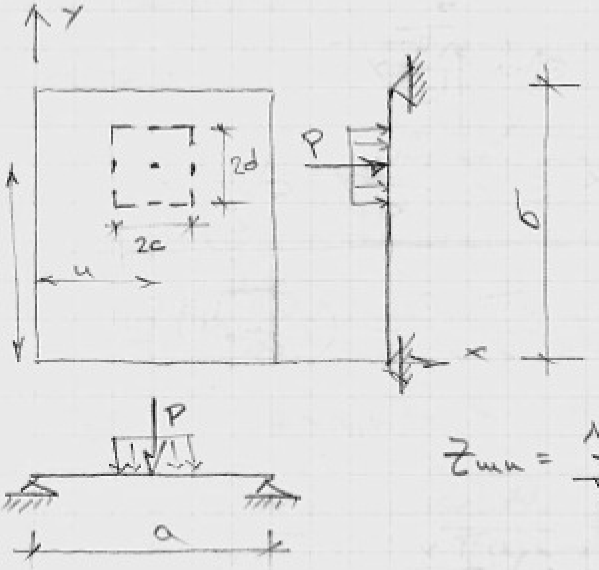
$$w_{max}=w\left(\frac{a}{2},\frac{b}{2}\right)=\frac{16z_0}{k\pi^6}\sum_m\sum_n\frac{(-1)^{\frac{m+n}{2}-1}}{mn\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}\ ;\quad m,n=1,3,5\dots$$

КВАДРАТНА ПЛОЧА $a/b=1$ $\quad m=n=1$

$$W_{max}=\frac{4z_0a^4}{k\pi^6}=0{,}00416\,\frac{z_0a^4}{k}$$

$m,n\to\infty$: $\quad w_{max}=0{,}00406\,\dfrac{z_0a^4}{k}$

2.



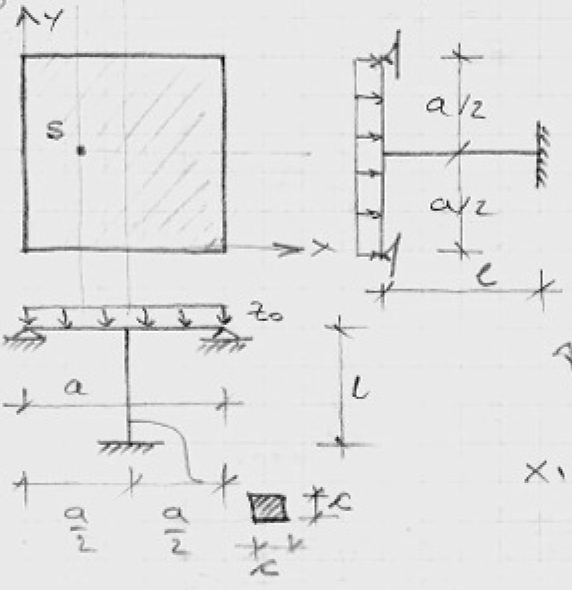
$$\sum_m\sum_n A_{mn}\sin\frac{m\pi}{a}x\,\sin\frac{n\pi}{b}y$$

$$A_{mn}=\frac{Z_{mn}}{K\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}$$

$$Z_{mn}=\frac{16Z_0}{\pi^2 mn}\sin\frac{m\pi u}{a}\sin\frac{m\pi c}{a}\sin\frac{n\pi v}{b}\cdot\sin\frac{n\pi d}{b}$$

$$w(x,y)=\frac{4P}{K\pi^4 ab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{\sin\frac{m\pi}{a}u\,\sin\frac{n\pi}{b}v}{\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2}\sin\frac{m\pi}{a}x\,\sin\frac{n\pi}{b}y$$

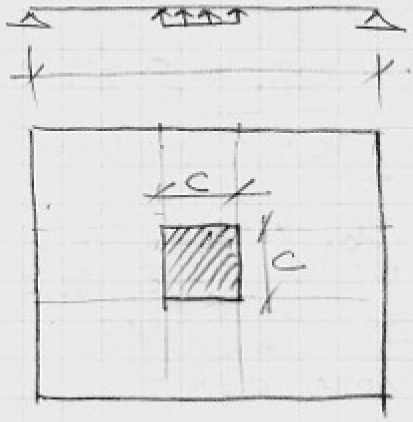
3. За плочу на скици одредити угиб и пресјечне силе у тачки S користећи први члан!



Релативно вертикално помјерање мора бити 0 на споју плоче и стуба!

$X_1=1$

$\delta_{11}^{пл.}\ (>0)$



$z_1=\frac{1}{c^2}$

$X_1=?$

$\delta_{11}X_1+\delta_{10}=0$

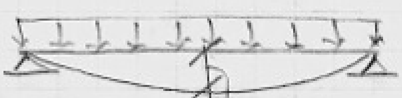
$\delta_{11}=\delta_{11}^{пл.}+\delta^{ст.}$

$\delta_{10}=\delta_{10}^{пл.}+\underbrace{\delta_{10}^{ст.}}_{0}$

$\delta_{11}^{ст.}=\frac{1}{EF}\cdot l$

$F = c^2$

$X_1=0 \quad z_0\neq 0$



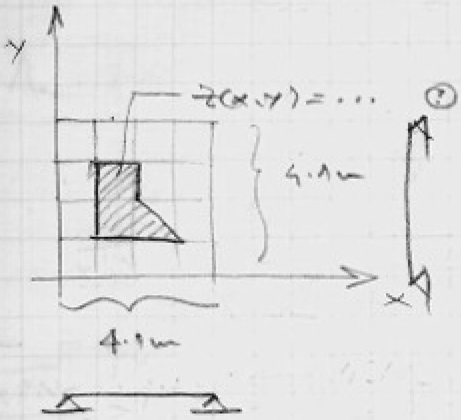
$\delta_{10}^{пл}=-\frac{4z_0a^2}{K\pi^6}$

$X_1=-\frac{\delta_{10}}{\delta_{11}}$

$w_0 = w_{s0} + w_{s1} x_1$

$M_{xs} = M_{xs0} + M_{xs0}\, x_1$

Домаћи:



$\bar{z}_0 = 20\,kN/m^2$

$d_p = 15\,cm$

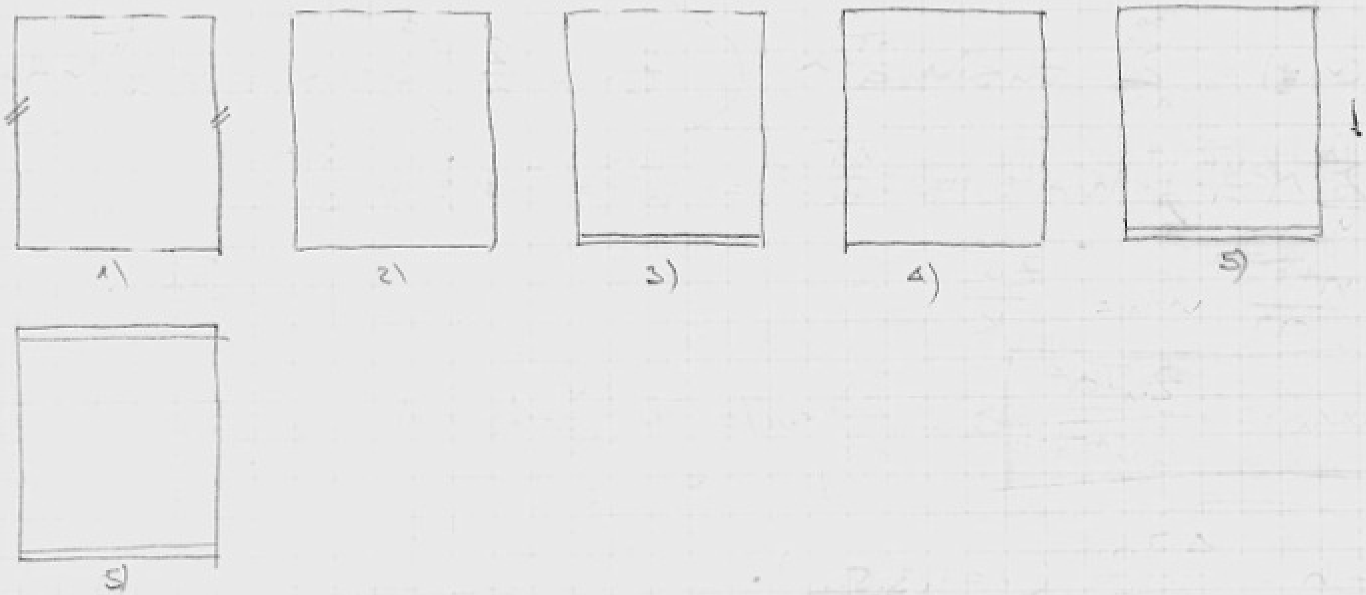
$E = 30\,GPa$

$\nu = 0{,}15$

Одредити: угиб и моменте савијања у средини плоче користити само први дио рјеш.

25.10.2005.

*M. LEVY-ово рјешење:



$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{z(x,y)}{K}$$

Претпостављамо да се z не мијења дуж y-осе

Партикуларно зависи само од спољашњег површинског оптерећења!

$w = w_0 + w_1$ — решење хомогене диф. једначине $\Delta\Delta w_1 = 0$

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$w_1 = ?$

$$\boxed{w_1 = \sum_{n=1}^{\infty} Y_n(y)\cdot \sin\frac{n\pi}{a} x}$$

$$\sum_{n=1}^{\infty}\left(Y_n^{IV} - 2\frac{n^2\pi^2}{a^2}Y_n'' + \frac{n^4\pi^4}{a^4}Y_n\right)\sin\frac{n\pi}{a}x = 0$$

$$\boxed{Y_n^{IV} - 2\frac{n^2\pi^2}{a^2}Y_n'' + \frac{n^4\pi^4}{a^4}Y_n = 0}$$

$$Y_n = \bar{A}_n e^{\frac{n\pi}{a}y} + \bar{B}_n e^{-\frac{n\pi}{a}y} + \bar{C}_n y e^{\frac{n\pi}{a}y} + \bar{D}_n y e^{-\frac{n\pi y}{a}}$$

$$\boxed{Y_n = \left(A_n + \frac{n\pi y}{a}B_n\right)\cdot ch\frac{n\pi y}{a} + \left(C_n + \frac{n\pi}{a}y D_n\right) sh\frac{n\pi y}{a}}$$

$w_0 = ?$

$$w_0 = \sum W_n \cdot \sin\frac{n\pi}{a}x$$

$$Z(x,y) = \sum_{n=1}^{\infty} Z_n \sin\frac{n\pi}{a}x \qquad Z_n = \frac{2}{a}\int_0^a Z(x,y)\sin\frac{n\pi}{a}x\,dx$$

$$\sum_n\left(\frac{n\pi}{a}\right)^4 W_n \sin\frac{n\pi}{a}x = \frac{1}{k}\sum_n Z_n \sin\frac{n\pi}{a}x$$

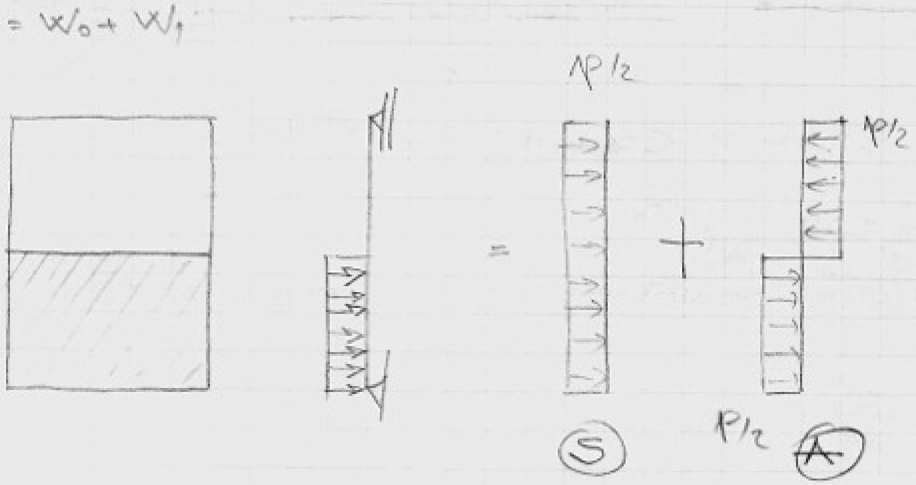
$$\frac{n^4\pi^4}{a^4}w_n = \frac{Z_n}{k}$$

$$\boxed{w_n = \frac{Z_n\cdot a^4}{k n^4\pi^4}} \qquad Z(x,y) = Z_0 = const$$

$$Z_n = \frac{4Z_0}{n\pi} \quad (n = 1,3,5\ldots)$$

$$w_0 = \sum_n \frac{4Z_0 a^4}{k n^5\pi^5}\sin\frac{n\pi}{a}x \; ; \; n = 1,3,5\ldots$$

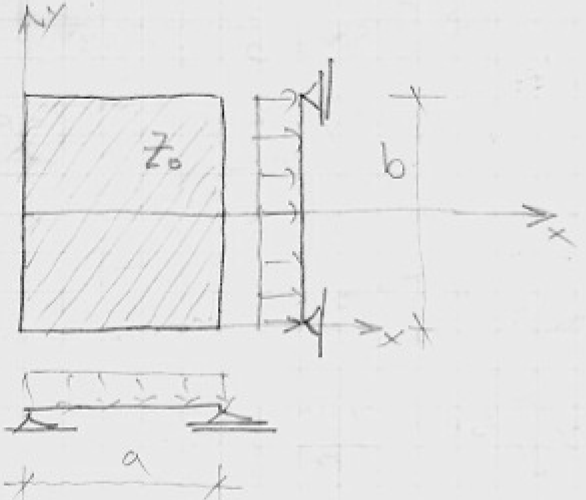
$w = w_0 + w_1$



(S): $w = w_0 + \sum_{n=1}^{\infty} \left(A_n \operatorname{ch}\frac{n\pi}{a}y + \frac{n\pi y}{a} D_n \cdot \operatorname{sh}\frac{n\pi y}{a}\right)\sin\frac{n\pi}{a}x$

(A): $w = w_0 + \sum_{n=1}^{\infty}\left(\frac{n\pi y}{a} B_n \operatorname{ch}\frac{n\pi}{a}y + C_n \operatorname{sh}\frac{n\pi}{a}y\right)\sin\frac{n\pi}{a}x$

A.



$$y = \pm\frac{b}{2}: \begin{cases} w = 0 \quad \ldots \quad (1) \\ M_y = 0 \Rightarrow \dfrac{\partial^2 w}{\partial y^2} = 0 \quad (2) \end{cases}$$

$$w = w_0 + w_1 = \sum_{n=1}^{\infty}\left(\frac{4 Z_0 \cdot a^4}{k\pi^5 n^5} + A_n \operatorname{ch}\frac{n\pi y}{a} + D_n \frac{n\pi y}{a}\operatorname{sh}\frac{n\pi y}{a}\right) \cdot$$
$$\cdot \sin\frac{n\pi}{a}x$$

$$\frac{\partial^2 w}{\partial y^2} = \sum\left(\frac{n\pi}{a}\right)^2\left(A_n \operatorname{ch}\frac{n\pi}{a}y + D_n\left(2\operatorname{ch}\frac{n\pi}{a}y + \frac{n\pi}{a}y\operatorname{sh}\frac{n\pi}{a}y\right)\right)$$
$$\cdot \sin\frac{n\pi}{a}x$$

(1): $\frac{4 Z_0 \cdot a^4}{k\pi^5 n^5} + A_n \operatorname{ch}\frac{n\pi}{2a}b + D_n \frac{n\pi}{2a}b\operatorname{sh}\frac{n\pi}{2a}b = 0$

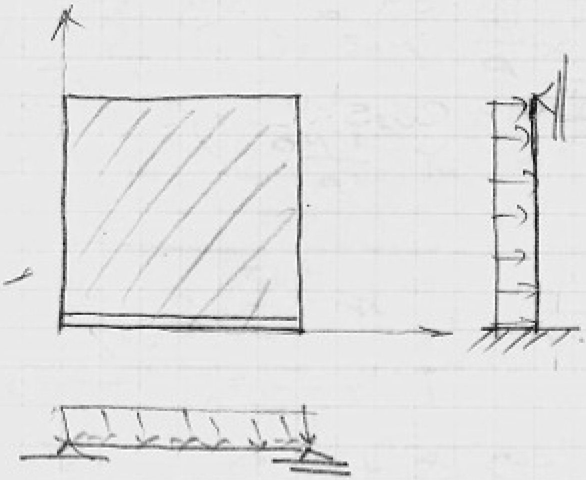
$\alpha_n = \frac{n\pi b}{2a}$ $\qquad S = \frac{4 \cdot Z_0 \cdot a^4}{k\pi^5}$

$$\boxed{A_n \operatorname{ch}\alpha_n + D_n \cdot \alpha_n \operatorname{sh}\alpha_n + \frac{S}{n^5} = 0} \quad (1)$$

(2) $$\boxed{A_n \operatorname{ch}\alpha_n + D_n\left(2\operatorname{ch}\alpha_n + \alpha_n \operatorname{sh}\alpha_n\right) = 0} \quad (2)$$

$$A_n = -\frac{1}{n^5}\,\frac{\alpha_n \operatorname{th}\alpha_n + 2}{2 \cdot \operatorname{ch}\alpha_n}\,S \qquad D_n = \frac{S}{n^5}\,\frac{1}{2\operatorname{ch}\alpha_n}$$

2.

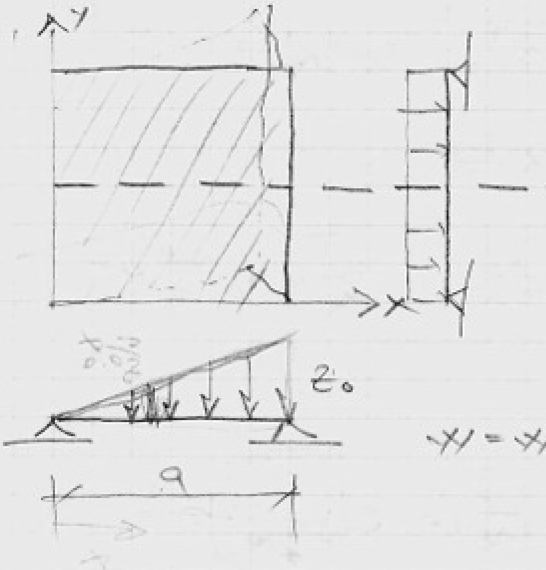


$$y=0:\ \begin{cases} w=0 & \ldots (1) \\ \dfrac{\partial w}{\partial y}=0 & \ldots (2) \end{cases}$$

$$y=b:\ \begin{cases} w=0 & \ldots (3) \\ M_y=0 \Leftrightarrow \dfrac{\partial^2 w}{\partial y^2}=0 & \ldots (4) \end{cases}$$

$$w = w_0 + w_1 = \sum_{n=1}^{\infty}\left[\frac{4Z_0 a^4}{k\pi^5 n^5} + \left(A_n + \frac{n\pi y}{a}B_n\right)\operatorname{ch}\frac{n\pi y}{a} + \left(C_n + \frac{n\pi}{a}y D_n\right)\operatorname{sh}\frac{n\pi}{a}y\right]\sin\frac{n\pi x}{a} \ \ldots$$

3.



$$w = w_0 + w_1 = w_0 + \sum_{n=1}^{\infty}\left(A_n \operatorname{ch}\frac{n\pi}{a}y + D_n\frac{n\pi y}{a}\cdot\operatorname{sh}\frac{n\pi}{a}y\right)\cdot\sin\frac{n\pi x}{a}$$

$$Z(x,y) = \frac{Z_0}{a}x\ ;\ Z(x,y) = \sum_n Z_n \sin\frac{n\pi}{a}x$$

$$Z_n = \frac{2}{a}\int_0^a \frac{Z_0}{a}x\sin\frac{n\pi}{a}x\,dx = \frac{2Z_0}{n\pi}(-1)^{n+1}$$

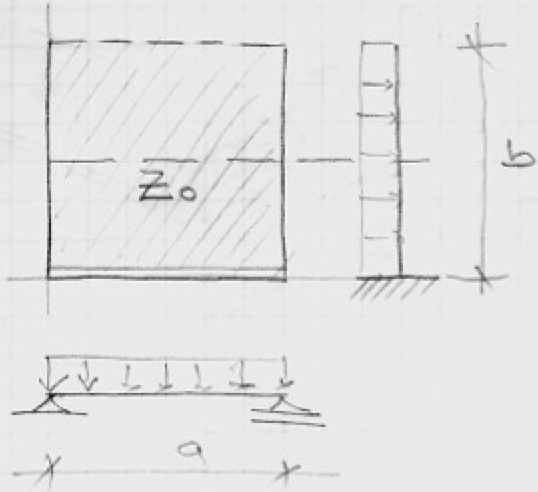
$$w_n = \frac{2Z_0\cdot a^4}{k n^5\pi^5}(-1)^{n+1} \qquad W_0 = \sum_n w_n\cdot\sin\frac{n\pi}{a}x$$

$$w = \sum_{n=1}^{\infty}\left[\frac{2Z_0\cdot a^4}{k n^5\pi^5}(-1)^{n+1} + A_n\operatorname{ch}\frac{n\pi}{a}y + D_n\frac{n\pi}{a}y\operatorname{sh}\frac{n\pi}{a}y\right)\cdot\sin\frac{n\pi}{a}x$$

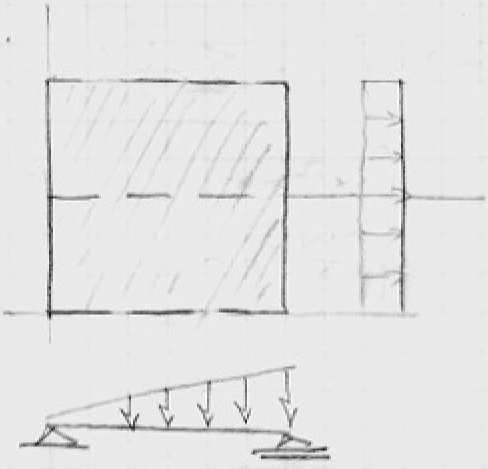
$$y=\pm\frac{b}{2}\ \begin{cases} w=0 \\ M_y=0 \Leftrightarrow \dfrac{\partial^2 w}{\partial y^2}=0 \end{cases}$$

Домаћи :

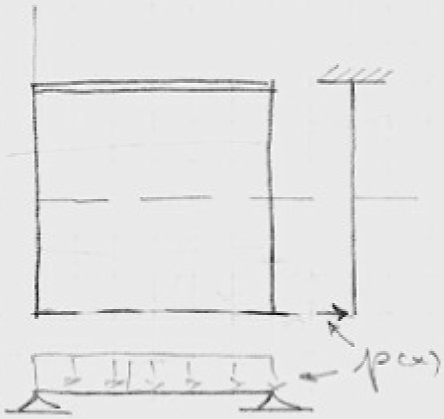
4.



5.



6.

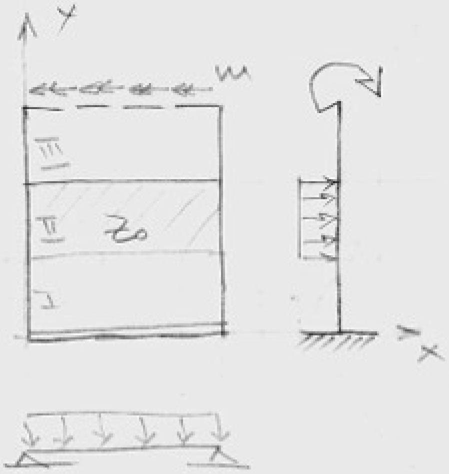


$$y=0 \quad \begin{cases} M_y = 0 \\ \overline{T}_y = -p(x) \end{cases}$$

$$y=b \quad \begin{cases} w = 0 \\ \dfrac{\partial w}{\partial y} = 0 \end{cases}$$

$$w = w_1 = \sum_{n=1}^{\infty} \left[\left(A_n + \frac{n\pi}{a} y \cdot B_n\right) ch \frac{n\pi}{a} y + \left(C_n + \frac{n\pi}{a} y D_n\right) \cdot sh \frac{n\pi y}{a} \right] \cdot \sin \frac{n\pi x}{a}$$

$$p(x) = \sum_{n=1}^{\infty} p_n \sin \frac{n\pi}{a} x; \quad p_n = \frac{2}{a} \int_0^a p(x) \sin \frac{n\pi}{a} x \, dx$$



$$w^{I} = w_1^{I} = \sum f(A_n^{I}, B_n^{I}, C_n^{I}, D_n^{I})$$

$$w^{II} = w_0 + w_1^{II} = \sum_n \left(\frac{4 Z_0 a^4}{k \pi^5 n^5} + A_n^{II}, B_n^{II}, C_n^{II}, D_n^{II} \right)$$

$$w''' = W_1''' = \sum f(A_n''', B_n''', C_n''', D_n''')$$

$$y=0: \begin{cases} w^I = 0 \\ \dfrac{\partial w^I}{\partial y} = 0 \end{cases} \qquad y = b: \begin{cases} M_y''' = -m \\ \bar{T}_y''' = 0 \end{cases}$$

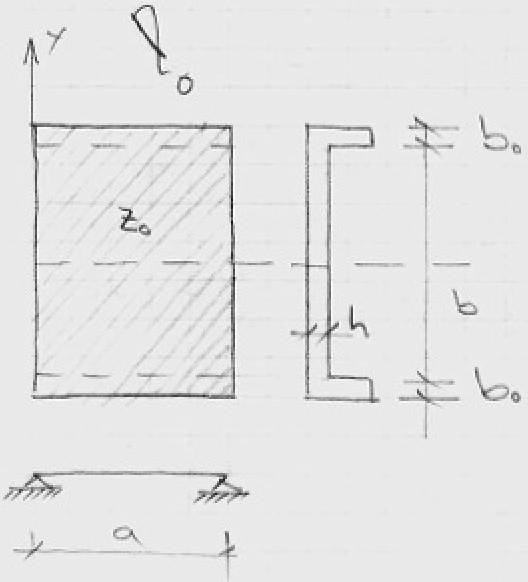
$$y = b_1: \begin{cases} w^I = w^{II} & M_y^I = M_y^{II} \\ \dfrac{\partial w^I}{\partial y} = \dfrac{\partial w^{II}}{\partial y} & \bar{T}_y^I = \bar{T}_y^{II} \end{cases}$$

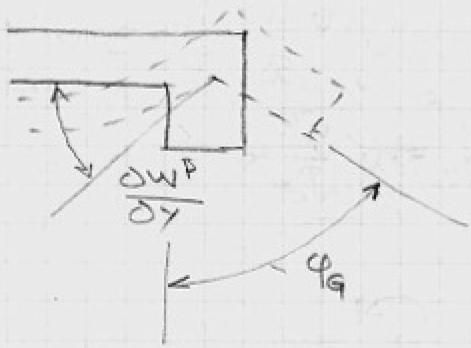
$$y = b_1 + b_2: \begin{cases} w^{II} = w^{III} & M_y^{II} = M_y^{III} \\ \dfrac{\partial w^{II}}{\partial y} = \dfrac{\partial w^{III}}{\partial y} & \bar{T}_y^{II} = \bar{T}_y^{III} \end{cases}$$

$$m = \sum_{n=1}^{\infty} m_n \sin \frac{n\pi}{a} x \qquad m_n = \frac{2}{a}\int_0^a m \sin \frac{n\pi}{a} x \, dx$$

1. XI 2005.

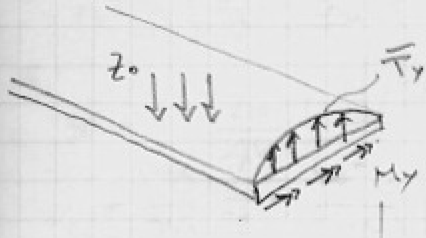
1.

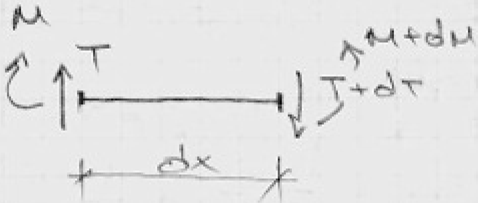




$$w = w_0 + w_1 = \sum_{n=1}^{\infty} \left(\frac{4 z_0 \cdot a^4}{k\pi^5 n^5} + A_n \operatorname{ch} \frac{n\pi y}{a} + \frac{n\pi y}{a} D_n \operatorname{sh} \frac{n\pi y}{a} \right) \sin \frac{n\pi}{a}$$

$$y = \pm \frac{b}{2} \begin{cases} w^P = w^G \quad (1) \iff \dfrac{\partial^4 w^P}{\partial x^4} = \dfrac{\partial^4 w^G}{\partial x^4} \\ \dfrac{\partial w^P}{\partial y} = \varphi_G \quad (2) \; / \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial x}\left(\dfrac{\partial w^P}{\partial y}\right) = \dfrac{\partial \varphi_G}{\partial x} = \theta_G \end{cases}$$





M_y $p = \overline{T}_y$ $\qquad \frac{dT}{dx} = -p = -\overline{T}_y \qquad \frac{dM}{dx} = T$

$$\frac{d^2M}{dx^2} = \frac{dT}{dx} = -\overline{T}_y$$

$$-EI_g \frac{d^4 w^g}{dx^4} = -\overline{T}_y$$

$$EI_g \frac{d^4 w^g}{dx^4} = k\left[\frac{\partial^3 w^p}{\partial y^3} + (2-\nu)\frac{\partial^3 w^p}{\partial x^2 \partial y}\right]$$

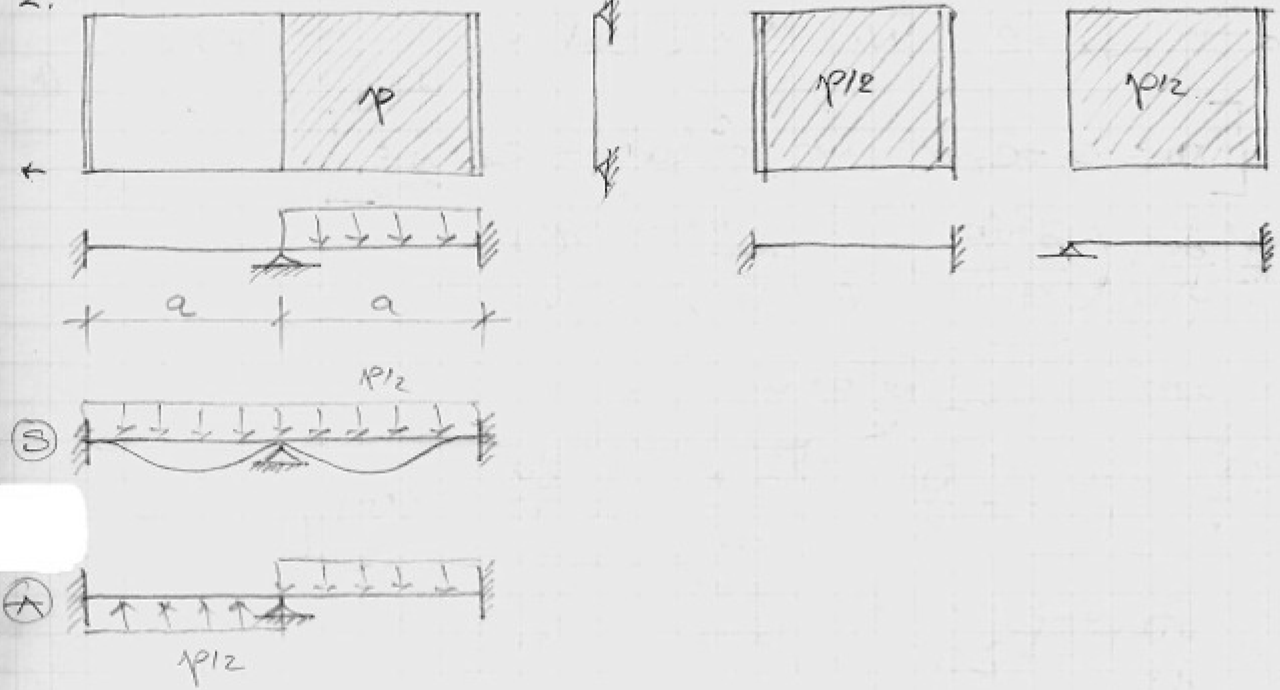
(1): $$\frac{\partial^4 w^p}{\partial x^4} = \frac{k}{EI_g}\left[\frac{\partial^3 w^p}{\partial y^3} + (2-\nu)\frac{\partial^3 w^p}{\partial x^2 \partial y}\right]$$

$$\frac{\partial M_t}{\partial x} = M_g \qquad \theta_g = \frac{M_t}{GI_t} \qquad \frac{\partial}{\partial x}\left(\frac{\partial w^p}{\partial y}\right) = \frac{M_t}{GI_t}$$

$$\frac{\partial}{\partial x}\left(GI_t \frac{\partial^2 w^p}{\partial x \partial y}\right) = \frac{\partial M_t}{\partial x} = M_y \qquad GI_t \cdot \frac{\partial^3 w^p}{\partial x^2 \partial y} = -k\left(\frac{\partial^2 w^p}{\partial y^2} + \nu \frac{\partial^2 w^p}{\partial x^2}\right)$$

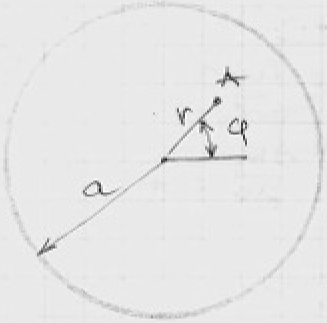
(2)

2.



САВИЈАЊЕ КРУЖНИХ ПЛОЧА:

$A(r,\varphi)$



$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\right)\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\right.$$

$$\left.+\frac{1}{r^2}\frac{\partial^2 w}{\partial\varphi^2}\right)=\frac{Z(r,\varphi)}{k}\qquad \Delta\Delta w=\frac{Z(r,\varphi)}{k}$$

ПРЕСЈ. СИЛЕ:

$$\Delta=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\qquad Z=Z(r)\Rightarrow w=w(r)\Rightarrow \text{ПРЕСЈ. СИЛЕ}(r)$$

$$\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)\left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}\right)=\frac{Z(r)}{k}$$

$$\boxed{\frac{d^4w}{dr^4}+\frac{2}{r}\frac{d^3w}{dr^3}-\frac{1}{r^2}\frac{d^2w}{dr^2}+\frac{1}{r^3}\cdot\frac{dw}{dr}=\frac{Z(r)}{k}}$$

$$w_0+w_1=w\qquad \Delta\Delta w=0\qquad r=e^t$$

$$\frac{d^4w}{dt^4}-4\frac{d^3w}{dt^3}+4\frac{d^2w}{dt^2}=0\qquad k^4-4k^3+4\cdot k^2=0$$

$$k_{1,2}=0\qquad k_{3,4}=2\qquad w_1=A+B\ln r+Cr^2+Dr^2\ln r$$

$$\left[\rho=\frac{r}{a}\right]\quad\left[w_1=C_1+C_2\cdot\rho^2+C_3\rho^2\cdot\ln\rho+C_4\ln\rho\right]\ \checkmark$$

$$\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)\underbrace{\left(\frac{d^2w_0}{dr^2}+\frac{1}{r}\frac{dw_0}{dr}\right)}_{-\frac{M}{k}}=\frac{Z(r)}{k}$$

$$\frac{d^2w_0}{dr^2}+\frac{1}{r}\frac{dw_0}{dr}=-\frac{M}{k}\qquad \boxed{\frac{d^2M}{dr^2}+\frac{1}{r}\frac{dM}{dr}=-Z(r)}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\cdot\frac{dM}{dr}\right)=-Z(r)\ /r/\int$$

$$r\cdot\frac{dM}{dr}=-\int rZ(r)\cdot dr\ /\frac{1}{r}/\int\qquad \boxed{M=-\int\frac{dr}{r}\int r\cdot Z(r)\,dr}\ \checkmark$$

$$\boxed{w_0=-\frac{1}{k}\int\frac{dr}{r}\int r\cdot M\,dr}\qquad Z(r)=Z_0$$

$$M = -\int \frac{dr}{r} \int z_0 \cdot r \cdot dr = -z_0 \cdot \int \frac{dr}{r} \int r dr = -z_0 \frac{r^2}{4}$$

$$w_0 = \frac{z_0}{4K} \int \frac{dr}{r} \int r^2 \cdot r \cdot dr = \frac{z_0}{4K} \int \frac{dr}{r} \cdot \frac{r^4}{4} = \frac{z_0 r^4}{64K} \qquad \rho = r/a$$

$$\boxed{w_0 = \frac{z_0 a^4}{64K} \rho^4}$$

$M_x, M_y, M_{xy}, T_x, T_y \quad (x, y) \qquad z = z(r)$

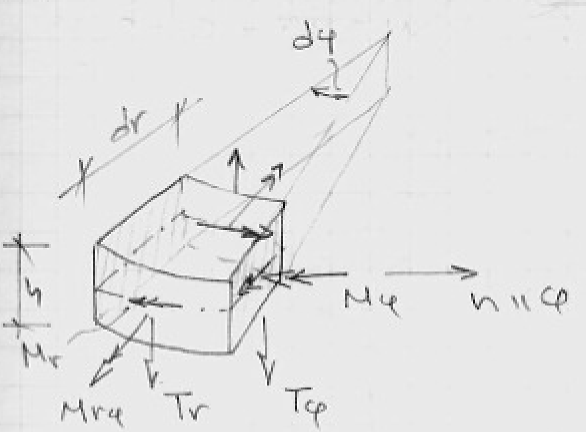
$M_r, M_\varphi, M_{r\varphi}, T_r, T_\varphi \quad (r, \varphi) \qquad M_r, M_\varphi, T_r \neq 0 \quad M_{r\varphi}, T_\varphi \equiv 0$

(r)

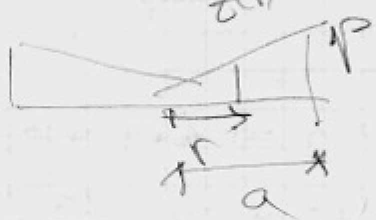
$$M_r = -K \left(\frac{d^2w}{dr^2} + \frac{\nu}{r} \frac{dw}{dr} \right)$$

$$M_\varphi = -K \left(\frac{1}{r} \frac{dw}{dr} + \nu \frac{d^2w}{dr^2} \right)$$

$$T_r = -K \frac{d}{dr} \left(\frac{d^2w}{dr^2} + \frac{1}{r} \frac{dw}{dr} \right)$$

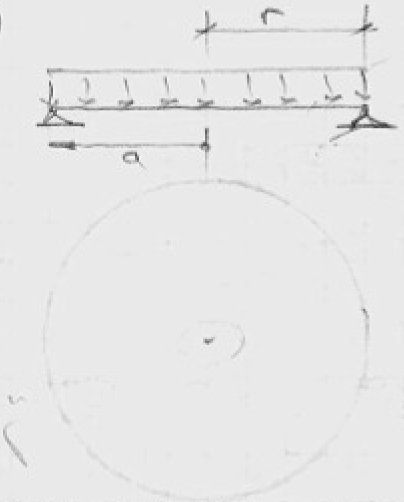


$$z(r) = \frac{p}{a} \cdot r$$



z_0

1)



написати израз за угиб

$$\underline{w = w_0 + w_1} = \frac{z_0 a^4}{64K} (\rho^4 + c_1 + c_2\rho^2 + c_3\rho^2 \ln\rho + c_4 \ln\rho)$$

$r = a$

$\rho = 1$; $\begin{cases} w = 0 & \dots (1) \\ M_r = 0 & \dots (2) \end{cases}$ губе се константе $\ln\rho = -\infty$

$$\boxed{w = \frac{z_0 a^4}{64K} \left(\rho^4 + c_1 + c_2 \cdot \rho^2 \right)} = 0 \qquad \frac{z_0 a^4}{64K} (1 + c_1 + c_2) = 0 \qquad c_1 + c_2 = -1$$

$$M_r = -K \left(\frac{d^2w}{dr^2} + \frac{\nu}{r} \frac{dw}{dr} \right) \qquad \frac{dw}{dr} = \frac{dw}{d\rho} \cdot \frac{d\rho}{dr} = \frac{1}{a} \frac{dw}{d\rho}$$

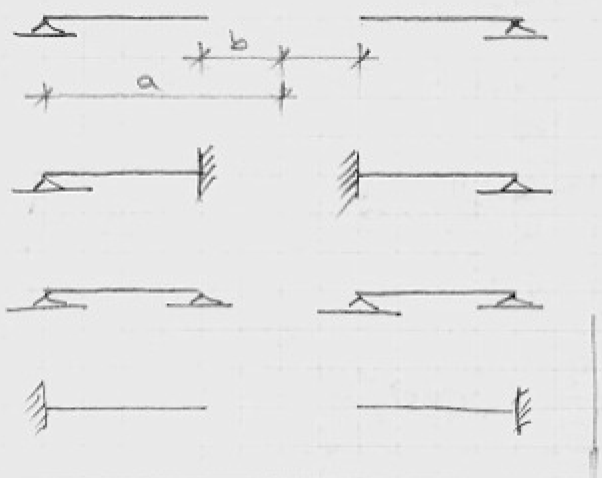
$$\frac{d^2w}{dr^2} = \frac{1}{a^2} \frac{d^2w}{d\rho^2} \qquad M_r = -\frac{z_0 a^2}{16} \left[(3+\nu)\rho^2 + \frac{1+\nu}{2} c_2 \right] \qquad w =$$

$$\rho = 1 \quad M_2 = 0 \Rightarrow \boxed{c_2 = -\frac{2(3+\nu)}{1+\nu}} \Longrightarrow \boxed{c_1 = \frac{2(3+\nu)}{1+\nu} - 1}$$

$$w = \frac{z_0 a^4}{16k}\left[(1-\rho^2)^2 + \frac{4(1-\rho^2)}{1+\nu}\right] \qquad M_r = \frac{z_0 \cdot r^2}{16}\cdot(3+\nu)(1-\rho^2)$$

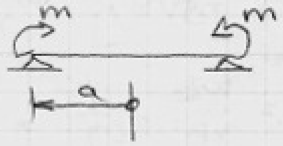
$$M_\varphi = \frac{z_0 a^2}{16}\left[(3+\nu) - \rho^2(1+3\nu)\right] \qquad T_r = -\frac{z_0 a}{2}\rho$$

ПРСТЕНАСТЕ ПЛОЧЕ:



$$w = w_0 + w_1 = w_0 + c_1 + c_2 r^2 + c_3 \ln r \cdot r^2 + c_4 \ln r$$

$$r=b: \begin{cases} M_r = 0 \\ T_r = 0 \end{cases} \qquad r=a \begin{cases} w=0 \\ \cancel{\frac{dw}{dr}=0} \end{cases}$$

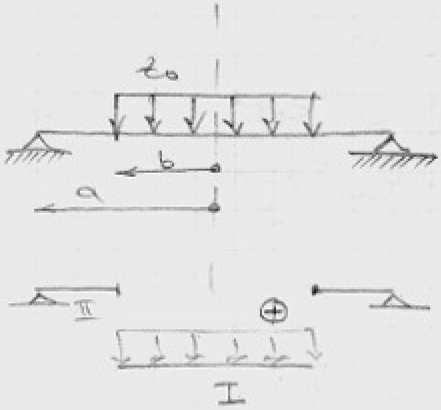


$$w = w_1 = c_1 + c_2\rho^2$$

$$\begin{matrix} r=a \\ \rho=1 \end{matrix} \begin{cases} W=0 \Rightarrow C_1 + C_2 = 0 \\ M_r = m \Rightarrow -\frac{2k}{a^2}(1+\nu)C_2 = m \end{cases}$$

$$C_2 = -\frac{ma^2}{2k(1+\nu)}(1-\rho^2) \qquad M_z = M_\varphi = m \qquad T_r = 0$$ – чисто савијање

2)



$$\rho = r/a$$

$$w = w_0 + w_1 = \frac{z_0 a^4}{64k}(\rho^4 + c_1^{I} + c_2^{I})$$

$$w_2 = c_1^{II} + c_2^{II} * \rho^2 + c_3^{II}\rho^2\ln\rho + c_4^{II}\ln\rho$$

$$\begin{matrix} r=a \\ \rho=1 \end{matrix} \begin{cases} w_2 = 0 & (1) \\ M_r^{II} = 0 & (2) \end{cases} \qquad \begin{matrix} r=b. \\ \rho=\frac{b}{a} \end{matrix} \begin{cases} w_1 = w_2 \\ \frac{dw_1}{dr} = \frac{dw_2}{dr} \\ M_r^{I} = M_r^{II} \\ T_1 = T \end{cases}$$

$$\frac{d^2 w_1}{dr^2} = \frac{d^2 w_2}{dr^2} \Leftrightarrow M_{r1} = M_{r2} \qquad \frac{d^3 w_1}{dr^3} = \frac{d^3 w_2}{dr^3} \Leftrightarrow T_{r1} = T_{r2}$$

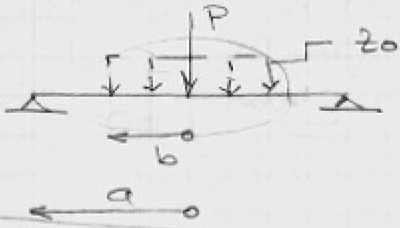
$$0 \le \rho \le \frac{b}{a} = \beta:$$

$$w' = \frac{z_0 b^2}{16k}\left\{\frac{a^4\rho^4}{4b^2} + a^2\rho^2\left[\frac{(1-\nu)b^2 - 4a^2}{2(1+\nu)a^2} + 2\ln\beta\right] + \frac{4(3+\nu)a^2 - (7+3\nu)b^2}{4(1+\nu)} + b^2\ln\beta\right\}$$

$\beta \le \rho \le 1$

$$w_2 = \frac{z_0 b^2}{16K}\left[\frac{3+\nu}{1+\nu}a^2(1-\rho^2) - 2a^2\rho^2\ln\frac{1}{\rho} - \frac{1-\nu}{2(1+\nu)}b^2(1-\rho^2) - b^2\ln\frac{1}{\rho}\right]$$

3)



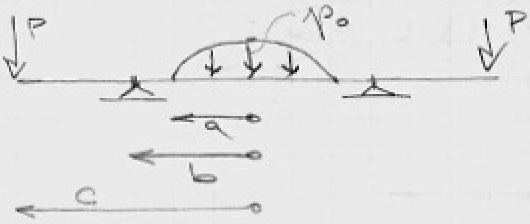
$$z_0 = \frac{P}{b^2\pi}$$

$$\lim_{x\to 0} x^2 =$$

$$\lim_{b\to 0} w_2 = w = \frac{P}{16K\pi}\left[\frac{3+\nu}{1+\nu}a^2(1-\rho^2) - 2a^2\rho^2\ln\frac{1}{\rho}\right]$$

$$M_r = \frac{P(1+\nu)}{4\pi}\ln\frac{1}{\rho} \quad\Big|\quad M_\varphi = \frac{P(1+\nu)}{4\pi}\left(\ln\frac{1}{\rho} + \frac{1-\nu}{1+\nu}\right)$$

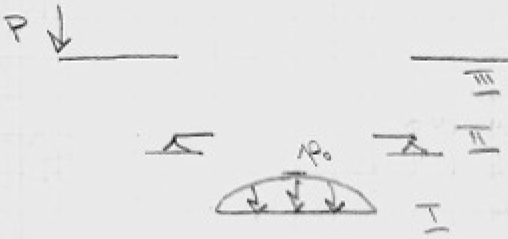
$\rho = 0$: $M_r, M_\varphi \to \infty$ $\quad \sigma_z = 0$



$$w_1 = w_0 + w_1 = w_0 + c_1' + c_2' r^2$$

$$w_2 = c_1'' + c_2'' r^2 + c_3 r^2 \ln r + c_4'' \ln r$$

$$w_3 = c_1'' + c_2'' r^2 + c_3 r^2 \ln r + c_4 \ln r$$



(3) (4)

$$r = a : \begin{cases} w_1 = w_2 \\ \dfrac{dw_1}{dr} = \dfrac{dw_2}{dr} \\ M_{r_1} = M_{r_2} \\ T_{r_1} = T_{r_2} \end{cases} \qquad r = c : \begin{cases} M_r = 0 \\ T_r = p \end{cases}$$

$$r = b \begin{cases} w_2 = 0 \\ w_3 = 0 \\ \dfrac{dw_2}{dr} = \dfrac{dw_3}{dr} \end{cases} \qquad M_{r_2} = M_{r_3}$$

КРУЖНА ПЛОЧА ПРИ ПРОИЗВОЉНОМ ОПТЕРЕЋЕЊУ 8.11.0

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\right)\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial\varphi^2}\right)=\frac{Z(r,\varphi)}{K}$$

$$w = w_0 + w_1 \qquad \Delta\Delta w_1 = 0$$

$$w_1 = W_0(r) + \sum_{m=1}^{\infty} W_m(r)\cdot\cos m\varphi + \sum_{m=1}^{\infty} \overline{W}_m(r)\cdot\sin m\varphi$$

$$\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\,\frac{m^2}{r^2}\right)\cdot\left(\frac{d^2W_m}{dr^2}+\frac{1}{r}\frac{dW_m}{dr}-\frac{m^2W_m}{r^2}\right)=0$$

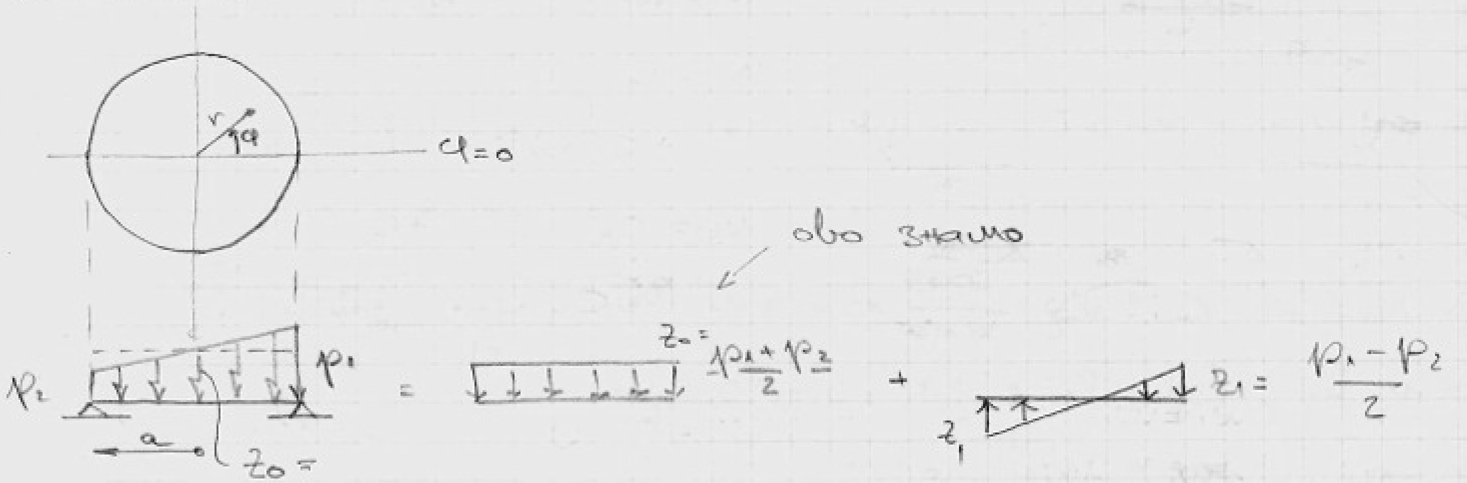
$$r = e^t$$

$$m=0: \quad w_1 = W_0(r) = A_0 + B_0 r^2 + C_0\ln r + D_0 r^2 \ln r$$

$$m=1: \quad w_1 = W_1 = A_1 r + B_1 r^3 + \frac{C_1}{r} + D_1 r\ln r$$

$$m>1 \qquad w_1 = A_m r^m + B_m r^{-m} + C_m r^{m+2} + D_m r^{-m+2}$$

ПРИМЈЕР:



$$Z_1(r,\varphi) = (p_1-p_2)/2\cdot\frac{r}{a}\cdot\cos\varphi$$

$$w = w_0 + w_1 \qquad w_0 = A_0\cdot\frac{p r^5}{a}\cos\varphi \qquad A_0 = \frac{1}{192K}$$

$$w_0 = \frac{p a^4}{192K}\cdot\rho^5\cos\varphi \qquad p = \frac{p_1-p_2}{2}$$

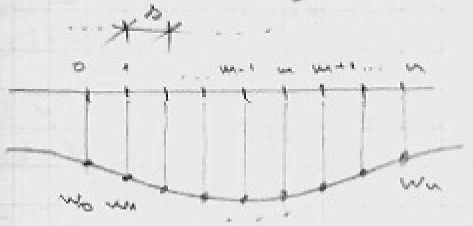
$m=1$

$$w_1 = A_1\rho + B_1\rho^3 + \frac{C_1}{\rho} + D_1\rho\ln\rho \qquad w = w_0 + w_1.$$

$$\rho = 1 \quad r = a \quad \left\{ \begin{array}{l} w = 0 \\ M_r = 0 \end{array} \right. \qquad \begin{array}{l} C_1 = 0 \\ D_1 = 0 \end{array}$$

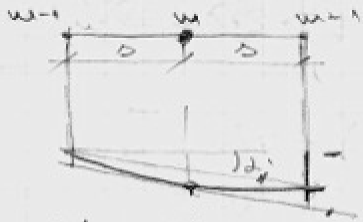
ДИФЕРЕНЦНИ ПОСТУПАК

Ово је нумерички поступак!



Диференцијална једначина -> диференцну једначину!

$$\left(\frac{dw}{dx}\right)_m = \operatorname{tg}\alpha \approx \boxed{\frac{w_{m+1} - w_{m-1}}{2\Delta}} \checkmark$$



$$\left(\frac{d^2w}{dx^2}\right)_m \approx \frac{\frac{w_{m+1} - w_m}{\Delta} - \frac{w_m - w_{m-1}}{\Delta}}{\Delta} = \frac{w_{m+1} - 2w_m + w_{m-1}}{\Delta^2}$$

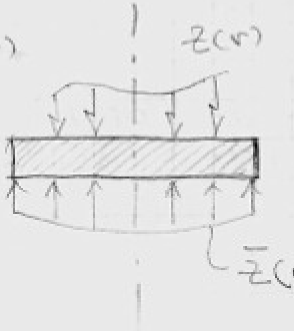
$$\left(\frac{d^3w}{dx^3}\right)_m \approx \frac{d}{dx}\left(\frac{d^2w}{dx^2}\right)_m \approx \frac{\left(\frac{d^2w}{dx^2}\right)_{m+1} - \left(\frac{d^2w}{dx^2}\right)_{m-1}}{2\Delta} =$$

$$= \frac{\frac{w_{m+2} - 2w_{m+1} + w_m}{\Delta^2} - \frac{w_m - 2w_{m-1} + w_{m-2}}{\Delta^2}}{2\Delta} = \frac{w_{m+2} - 2w_{m+1} + 2w_{m-1} - w_{m-2}}{2\Delta^3}$$

$$\left(\frac{d^4w}{dx^4}\right)_m = \frac{d^2}{dx^2}\left(\frac{d^2w}{dx^2}\right)_m \approx \frac{w_{m+2} - 4w_{m+1} + 6w_m - 4w_{m-1} + w_{m-2}}{\Delta^4}$$

ПРИМЈЕР

$w = w(r)$



$$\Delta\Delta w = \frac{z - \bar{z}}{k}$$

$$\frac{d^4w}{dr^4} + \frac{2}{r}\frac{d^3w}{dr^3} - \frac{1}{r^2}\frac{d^2w}{dr^2} + \frac{1}{r^3}\frac{dw}{dr} + \frac{c\cdot w}{k} = \frac{z}{k}$$

$$(1+\lambda_m) w_{m+2} - \left[2(2+\lambda_m) + \frac{\lambda_m^2}{2}(2-\lambda_m)\right] w_{m+1} +$$

$$+ \left(6 + 2\lambda_m^2 + \frac{c}{k}\Delta^4\right) w_m - \left[2(2-\lambda_m) + \frac{\lambda_m^2}{2}(2+\lambda_m)\right] w_{m-1} +$$

$$+ (1-\lambda_m) w_{m-2} = \frac{z_m \Delta^4}{k}$$

$\Delta =$

$$\lambda_m = \frac{\Delta}{r_m}$$

$$M_{r,m} \cong -\frac{K}{\Delta^2}\left[w_{m+1}\cdot\left(1 + \frac{\nu\cdot\lambda_m}{2}\right) - 2\cdot w_m + w_{m-1}\left(1 - \frac{\nu\lambda_m}{2}\right)\right]$$

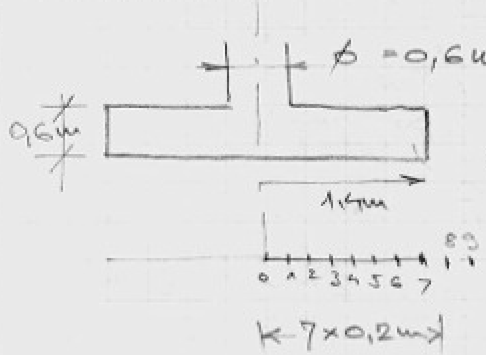
$$M_{\varphi,m} \cong -\frac{K}{\Delta^2}\left[w_{m+1}\left(\nu + \frac{\lambda_m}{2}\right) - 2\nu w_m + w_{m-1}\left(\nu - \frac{\lambda_m}{2}\right)\right]$$

$$T_{r,m} \approx -\frac{K}{2\Delta^3}\left[w_{m+2} - w_{m+1}\left(2 - 2\lambda_m + \lambda_m^2\right) - 4\lambda_m w_m + \right.$$

$$\left. + w_{m-1}\left(2 + 2\lambda_m + \lambda_m^2\right) - w_{m-2}\right]$$

ПРИМЈЕР

$P = 150$ kN



$\Delta = 0{,}2$ m

$\nu = 0{,}16$

$E = 2{,}1\cdot 10^7 \ \frac{kN}{m^2}$

$C = 10^5$ kN/m³

$K = 3{,}888\cdot 10^5$

$p = \frac{P}{0{,}3^2\pi} = 5305{,}16 \ \frac{kN}{m^2}$

$r = a = 1{,}4$ m

$$\begin{cases} M_{r,7} = 0 \Rightarrow w_8 = 1{,}97648\cdot w_7 - 0{,}975480\, w_6 \\ T_{r,7} = 0 \Rightarrow w_9 = 4\cdot w_7 - 4 w_6 + w_5 \end{cases}$$

$m = 0 \quad \lambda_0 = \frac{\Delta}{r_0} = \frac{0{,}2}{0} \to \infty$

$$w = w(0) + \frac{r}{1!}\cancel{\left(\frac{dw}{dr}\right)_0}^{\to 0} + \frac{r^2}{2!}\left(\frac{d^2w}{dr^2}\right)_0 + \frac{r^3}{3!}\cancel{\left(\frac{d^3w}{dr^3}\right)_0}^{\to 0} +$$

$$+ \frac{r^4}{4!}\left(\frac{d^4w}{dr^4}\right)_0 + \ldots$$

$$w = w(0) + \frac{r^2}{2!}\left(\frac{d^2w}{dr^2}\right)_0 + \frac{r^4}{4!}\left(\frac{d^4w}{dr^4}\right)_0 + \dots$$

$$\frac{dw}{dr} \approx r\left(\frac{d^2w}{dr^2}\right)_0 + \frac{r^3}{3!}\left(\frac{d^4w}{dr^4}\right)_0 + \dots$$

$$\frac{d^2w}{dr^2} = \left(\frac{d^2w}{dr^2}\right)_0 + \frac{r^2}{2!}\left(\frac{d^4w}{dr^4}\right)_0 + \dots$$

$$M_r = -K\left(\frac{d^2w}{dr_m^2} + \frac{\nu}{r}\frac{dw}{dr_m}\right)$$

$$\frac{d^3w}{dr^3} = r\left(\frac{d^4w}{dr^4}\right)_0 + \dots$$

$$\frac{d^4w}{dr^4} = \left(\frac{d^4w}{dr^4}\right)_0 + \dots$$

$m=0$ $\quad \frac{8}{3}\left(\frac{d^4w}{dr^4}\right) + \frac{c}{K} w_0 = \frac{Z_0}{K}$

$$\frac{8}{3}\ \frac{w_2 - 4w_1 + 6w_0 - 4w_{-1} + w_{-2}}{\lambda^4} + \frac{c}{K} w_0 = \frac{Z_0}{K} \quad / \lambda^4$$

$$\boxed{\frac{16}{3} w_2 - \frac{64}{3} w_1 + \left(16 + \frac{c}{K}\lambda^4\right) w_0 = \frac{Z_0}{K}\lambda^4}$$

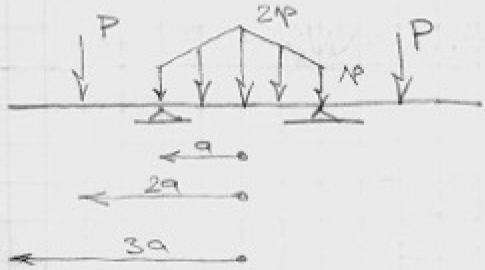
$m=1$

$\lambda_1 = \frac{\lambda}{r_1} = \frac{0{,}2}{0{,}2} = 1 \qquad 2 \cdot w_3 - 6{,}5\, w_2 + 8{,}000412\, w_1 - 3{,}5\, w_0 =$

$= 2{,}183 \cdot 10^{-5} \quad \Rightarrow$ 8 једначина! са 8 непозн.

$m=2$

1) ДОН:

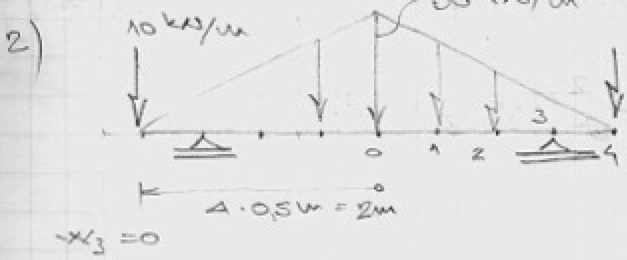


a) одредити партикуларно рјешење $w_0 = ?$

b) $w = ?$

c) Гранични услови

Прелазни услови



$E = 30$ GPa

$\nu = 0$

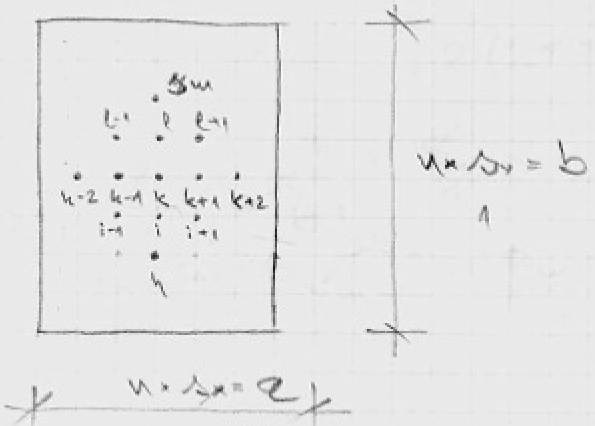
$h = 0{,}125$ m

$w_3 = 0$

Г.У.

15. 11. '05

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{z(x,y)}{k}$$



$$\left(\frac{\partial w}{\partial x}\right)_k \approx \frac{w_{k+1} - w_{k-1}}{2\cdot\Delta x}$$

$$\left(\frac{\partial^2 w}{\partial x^2}\right)_k \approx \frac{w_{k+1} - 2w_k + w_{k-1}}{\Delta x^2}$$

$$\left(\frac{\partial^3 w}{\partial x^3}\right)_k = \frac{w_{k+2} - 2w_{k+1} + 2w_{k-1} - w_{k-2}}{2\delta x^3}$$

$$\left(\frac{\partial^4 w}{\partial x^4}\right)_k = \frac{w_{k+2} - 4w_{k+1} + 6w_k - 4w_{k-1} + w_{k-2}}{\delta x^4}$$

$$\left(\frac{\partial w}{\partial y}\right)_k \approx \frac{w_l - w_i}{2\Delta y} \qquad \left(\frac{\partial^2 w}{\partial y^2}\right)_k \approx \frac{w_l - 2w_k + w_i}{\Delta y^2}$$

$$\left(\frac{\partial^3 w}{\partial y^3}\right)_k \approx \frac{w_m - 2w_l + 2w_i - w_n}{2\cdot\Delta y^3} \qquad \left(\frac{\partial^4 w}{\partial y^4}\right)_k \approx \frac{w_m - 4w_l + 6w_k - 4w_i}{\delta y^4}$$

$$\left(\frac{\partial^2 w}{\partial x \partial y}\right)_k = \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x}\right)_k \approx \frac{\left(\frac{\partial w}{\partial x}\right)_l - \left(\frac{\partial w}{\partial x}\right)_i}{2\cdot\delta y} = \frac{\frac{w_{l+1} - w_{l-1}}{2\cdot\delta x} - \frac{w_{i+1} - w_{i-1}}{2\delta x}}{2\cdot\delta y} =$$

$$= \frac{w_{l+1} - w_{l-1} - w_{i+1} + w_{i-1}}{4\cdot\delta x\delta y}$$

$$\left(\frac{\partial^4 w}{\partial x^2 \partial y^2}\right)_k = \frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 w}{\partial x^2}\right)_k = \frac{\left(\frac{\partial^2 w}{\partial x^2}\right)_l - 2\left(\frac{\partial^2 w}{\partial x^2}\right)_k + \left(\frac{\partial^2 w}{\partial x^2}\right)_i}{\delta y^2} =$$

$$= \frac{4w_k - 2(w_l + w_i + w_{k+1} + w_{k-1}) + w_{l+1} + w_{l-1} + w_{i+1} + w_{i-1}}{\delta x^2 \delta y^2}$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{z(x,y)}{k} \Big/ \alpha^2 \delta x^4$$

$$w_k\left[6\left(\alpha^2 + \frac{1}{\alpha^2}\right) + 8\right] - 4\cdot\left[(1+\alpha^2)\cdot(w_{k+1} + w_{k-1}) + \left(1 + \frac{1}{\alpha^2}\right)(w_l + w_i)\right] +$$

$$+ 2\cdot(w_{i-1} + w_{l-1} + w_{i+1} + w_{l+1}) + \alpha^2(w_{k+2} + w_{k-2}) +$$

$$+ \frac{1}{\alpha^2}(w_m + w_n) = \frac{z_k \cdot \alpha^2 \cdot \delta x^4}{k}$$

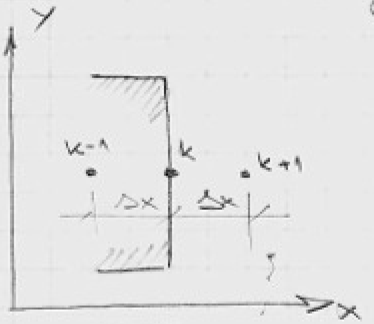
$\alpha = \frac{S_y}{S_x}$

$$\begin{array}{ccccc}
 & & \frac{1}{\alpha^2} & & \\
 & 2 & -4\left(1+\frac{1}{\alpha^2}\right) & 2 & \\
\alpha^2 & -4(1+\alpha^2) & 6\left(\alpha^2+\frac{1}{\alpha^2}\right)+8 & -4(1+\alpha^2) & \alpha^2 \\
 & 2 & -4\left(1+\frac{1}{\alpha^2}\right) & 2 & \\
 & & \frac{1}{\alpha^2} & &
\end{array}$$

$S_x = S_y \Rightarrow \alpha = 1$

$$\begin{array}{ccccc}
 & & 1 & & \\
 & 2 & -8 & 2 & \\
1 & -8 & 20 & -8 & 1 \\
 & 2 & -8 & 2 & \\
 & & 1 & &
\end{array}$$

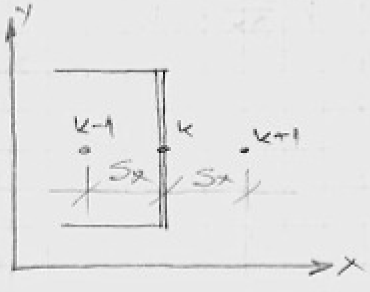
$+W_k$



Слободно ослоњен крај:

$W_k = 0$; $M_{x,k} = 0 \Leftrightarrow \left(\frac{\partial^2 W}{\partial x^2}\right)_k = 0 \Leftrightarrow \frac{W_{k+1} - 2W_k + W_{k-1}}{\Delta x^2} = 0$

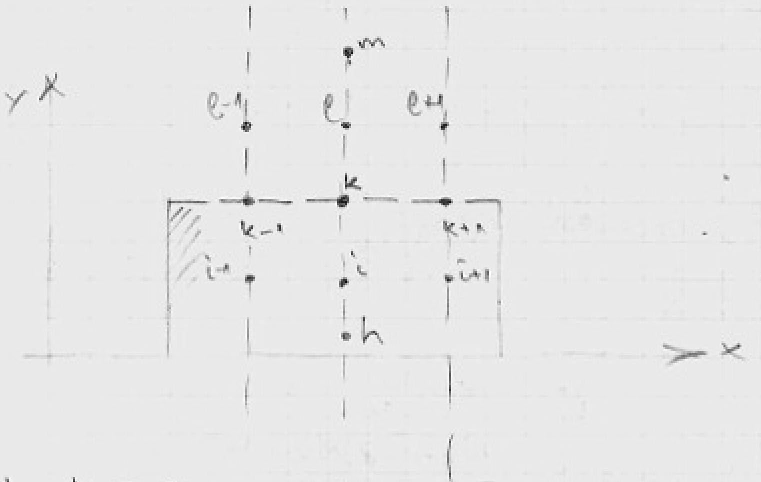
$W_{k+1} = -W_{k-1}$



Укљештен крај:

$W_k = 0 \qquad \left(\frac{\partial W}{\partial x}\right)_k = 0 \qquad \frac{W_{k+1} - W_{k-1}}{2 \cdot \Delta x} = 0 \quad \boxed{W_{k+1} = W_{k-1}}$

Плоча која има слободну ивицу:



$M_{y,k} = 0$

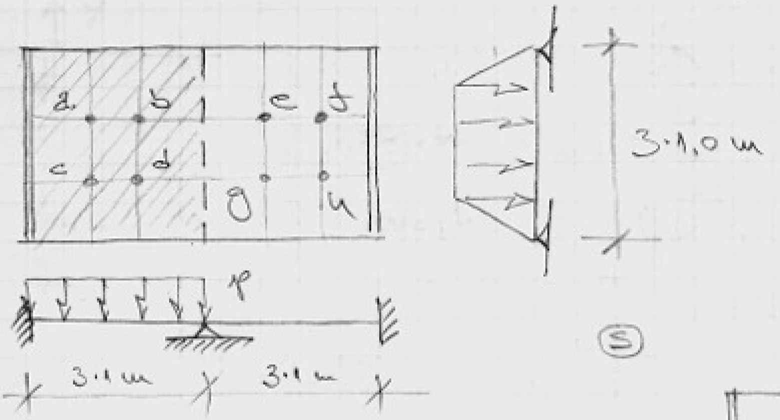
$\overline{T}_y = \left(T_y + \frac{\partial M_{xy}}{\partial x}\right)_k = 0 \qquad M_{y,k} = -K\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)_k$

$\frac{w_e - 2w_k + w_i}{\Delta y^2} + \nu \cdot \frac{w_{k+1} - 2w_k + w_{k-1}}{\Delta x^2} = 0 \;/ \cdot \Delta y^2$

$w_e = 2w_k - w_i + \nu \lambda^2 \left(-w_{k+1} + 2w_k - w_{k-1}\right)$

$-K \cdot \left[\frac{\partial^3 w}{\partial y^3} + \frac{\partial^3 w}{\partial x^2 \partial y} + (1-\nu)\frac{\partial^2 w}{\partial y^2}\right]_k = 0$

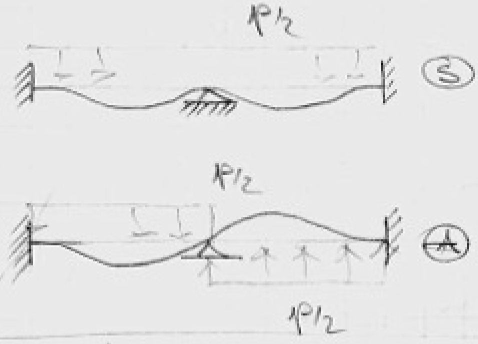
1.
а) Ријешити проблем савијања дате плоче и срачунати угибе у тачкама користећи само први члан реда усвојеног рјешења;
b) Диференцним поступком наћи угибе у означеним тачкама!

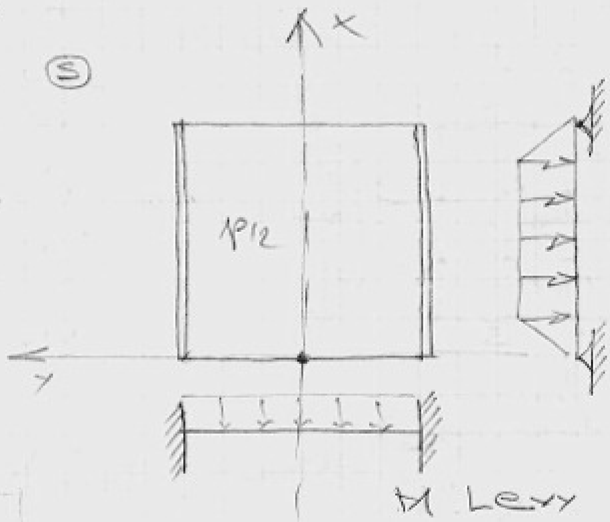


$E = 30$ GPa

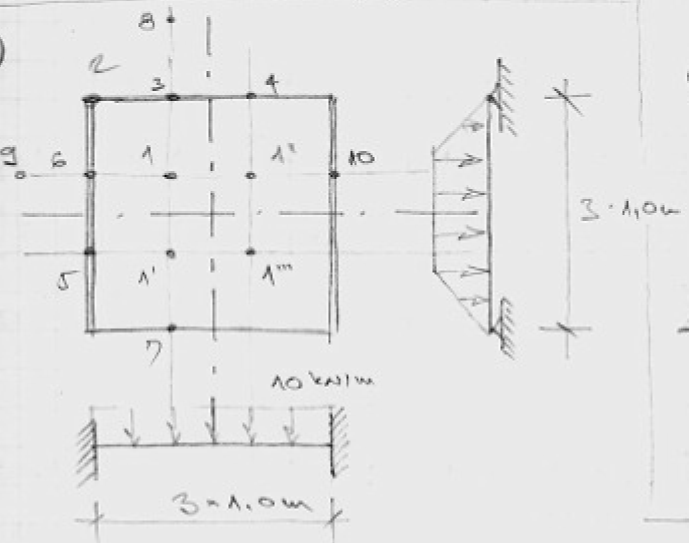
$h = 0{,}2$ m

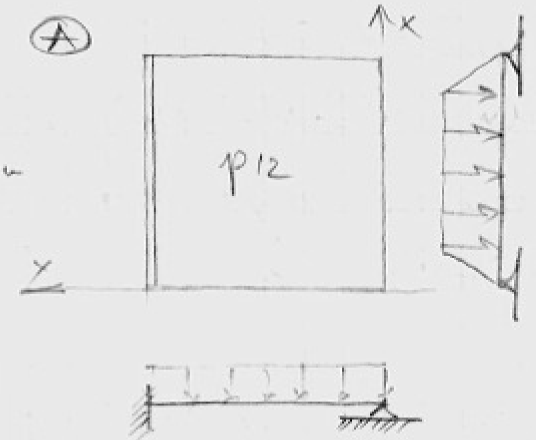
$\nu = 0{,}15$





b)





$w_1 = ?$

$\left(\frac{\partial w}{\partial x}\right)_c = 0 \quad : \quad w_9 = w_1 \qquad \Delta x = \Delta y \Rightarrow \alpha = 1 \qquad k = 1$

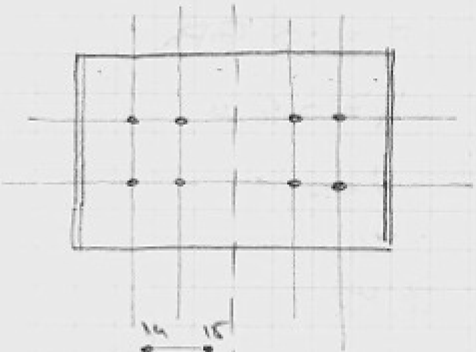
$w_2 = w_3 = w_4 = w_5 = w_6 = w_7 = w_{10} = 0$

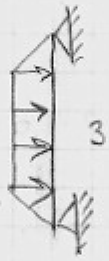
$M_{y3} = 0 \quad : \quad w_8 = -w_1$

$$20 w_1 - 8(\overset{= w_1}{w_1''} + \overset{0}{w_6} + \overset{0}{w_3} + \overset{= w_1}{w_1'}) + 2(\overset{0}{w_4} + \overset{0}{w_2} + \overset{= w_1}{w_1'''} + \overset{0}{w_5}) +$$

$$\underset{-w_1}{w_8} + \underset{0}{w_7} + \underset{0}{w_{10}} + \underset{w_1}{w_9} = 10/k$$

$6 \cdot w_1 = \frac{10}{k} \qquad w_1 = \frac{5}{3k}$





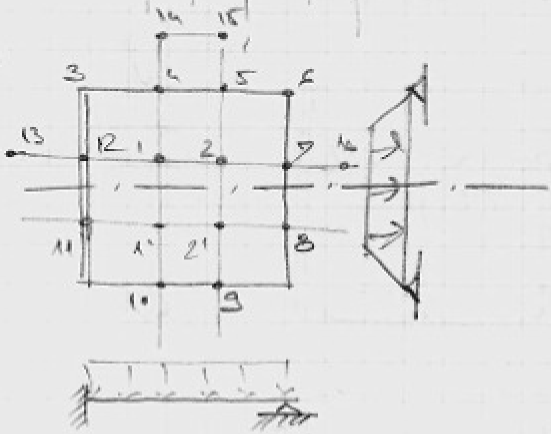
3·1,0

$w_1' = w_1$

$w_2' = w_2$

$w_3 = w_4 = \dots = w_{12} = 0$

A



$\left(\frac{\partial w}{\partial x}\right)_{12} = 0 \qquad w_{13} = w_1$

$M_{y,4} = 0 \Rightarrow w_{14} = -w_1$

$M_{y,5} = 0 \Rightarrow w_{15} = -w_2$

$M_{x,7} = 0 \Rightarrow w_{16} = -w_2$

$k = 1$

$20w_1 - 8(w_1 + w_{12} + w_4 + w_1') + 2(w_5 + w_3 + w_2' + w_{11}) + w_{14} + w_{10} + w_{1} +$

$= 10/k$

$12w_1 - 6w_2 = 10/k$

$k=2$:

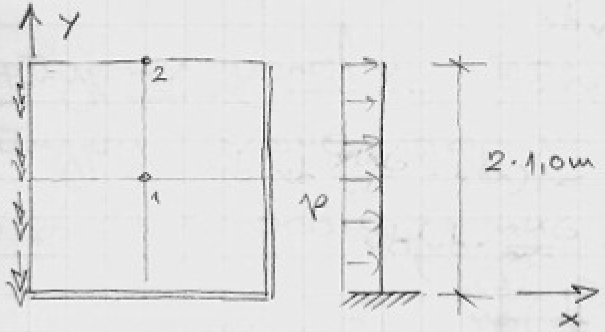
$20w_2 - 8(w_7 + w_1 + w_5 + w_2') + 2(w_6 + w_4 + w_8 + w_1') + w_{16} +$

$+ w_{12} + w_{15} + w_3 = 10/k$

$-6w_1 + 10w_2 = 10/k \qquad w_1 = \frac{1,90476}{K} \qquad w_2 = \frac{2,14286}{k}$

$w_a = w_1^S + w_1^A$

$w_b = w_1^S - w_2^A$

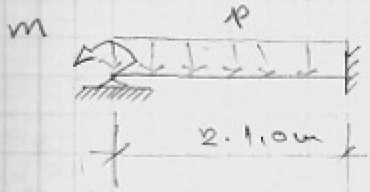


$E = 30\,GPa$

$\nu = 0$

$p = 20\ kN/m^2$

$m = 10\ kNm/m$



$x=0:\quad w=0$

$M_x = -m$

Плоче напрегнуте у својој равни

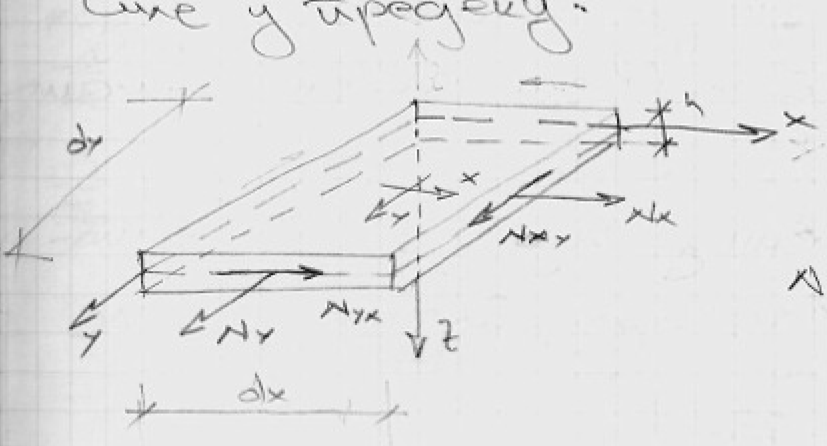
22.11.2005.

Напрезање у равни изазива напрезање које

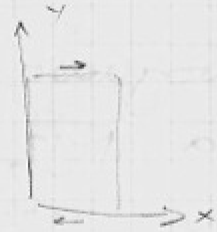
$+W_{13}=$ је паралелно са средњом равни плоче.

Стањ. оптерећење је равномјерно расподјељено по дебљини плоче

Силе у пресјеку.



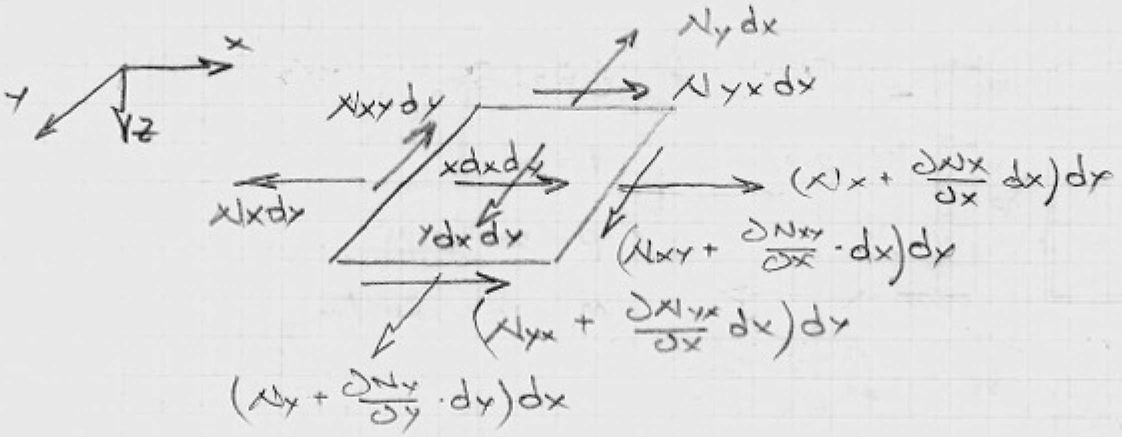
$N_{xy} = N_{yx}$



$N_x = h \cdot \sigma_x\ [kN/m]$

$N_y = h \cdot \sigma_y\ [kN/m]$

$N_{xy} = h \cdot \tau_{xy}\ [kN/m]$



1) $\sum X = 0: \quad \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} + X = 0$

2) $\sum Y = 0: \quad \frac{\partial N_{yx}}{\partial x} + \frac{\partial N_y}{\partial y} + Y = 0$

3) $\sum M_z = 0: \quad N_{xy} = N_{yx}$

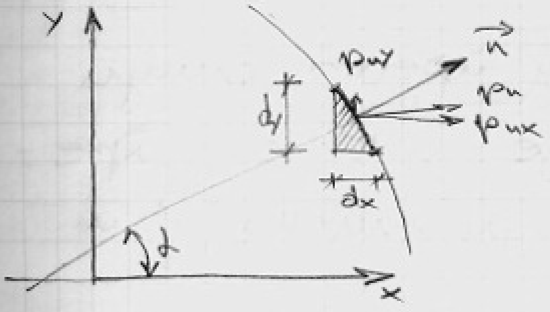
Од практичног значаја су нам само прва два услова пошто нам је трећи услов познат из става о конјугованости смичућих напона.
Имамо 3 непознате, а два услова. Проблем је стат. неодређен, па ће се дописати један услов компатибилности (веза између дилатације и клизања).

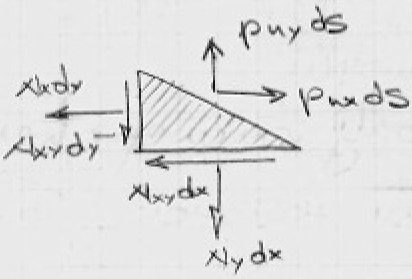
Уводимо нову величину која се зове напонска ф-ја. Њени изводи представљају силе у пресеку!

$$N_{xy} = -\frac{\partial^2 F}{\partial x \partial y} \qquad N_x = \frac{\partial^2 F}{\partial y^2} + \dots \qquad N_y = \frac{\partial^2 F}{\partial x^2} + \dots$$

$$\frac{\partial^4 F}{\partial x^4} + 2 \cdot \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0$$

<u>Контурни услови</u>: посматрамо произвољну контуру плоче. Нормала контуре заклапа угао α са x-осом.





$$\sin\alpha = -\frac{dx}{ds}$$

$$\cos\alpha = -\frac{dy}{ds}$$

$$\Sigma X = 0: \quad N_x ds \cdot \cos\alpha + N_{xy} ds \cdot \sin\alpha = p_{nx} ds \quad /:ds$$

$$\Sigma Y = 0 \quad N_y ds \sin\alpha + N_{xy} ds \cos\alpha = p_{ny} ds \quad /:ds$$

$$N_x \cos\alpha + N_{xy} \cdot \sin\alpha = p_{nx}$$

$$N_{xy} \cos\alpha + N_y \cdot \sin\alpha = p_{ny}$$

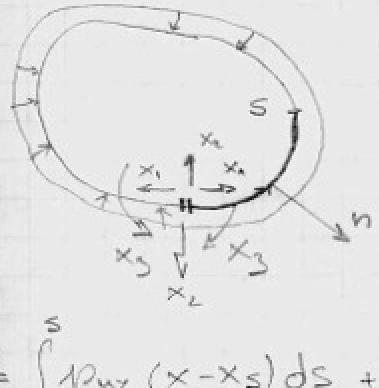
$$\frac{\partial^2 F}{\partial y^2} \cdot \frac{dy}{ds} + \frac{\partial^2 F}{\partial x \partial y} \cdot \frac{dx}{ds} = p_{nx}$$

$$-\frac{\partial^2 F}{\partial x \partial y} \cdot \frac{dy}{ds} - \frac{\partial^2 F}{\partial x^2} \frac{dx}{ds} = p_{ny}$$

$$\frac{\partial}{\partial s} \cdot \left(\frac{\partial F}{\partial y}\right) = p_{nx} \quad //\int \qquad \frac{\partial F}{\partial y} = \int_0^s p_{nx} ds + \left(\frac{\partial F}{\partial y}\right)_0$$

$$\frac{\partial}{\partial s} \left(\frac{\partial F}{\partial x}\right) = -p_{ny} \quad //\int \qquad \frac{\partial F}{\partial x} = -\int_0^s p_{ny} ds + \left(\frac{\partial F}{\partial x}\right)_0$$

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0$$



$$N_x = \frac{\partial^2 F}{\partial y^2} \quad N_y = \frac{\partial^2 F}{\partial x^2} \quad N_{xy} = -\frac{\partial^2 F}{\partial x \partial y}$$

$F = ?$ – на контури

$$dF = \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} \cdot dy$$

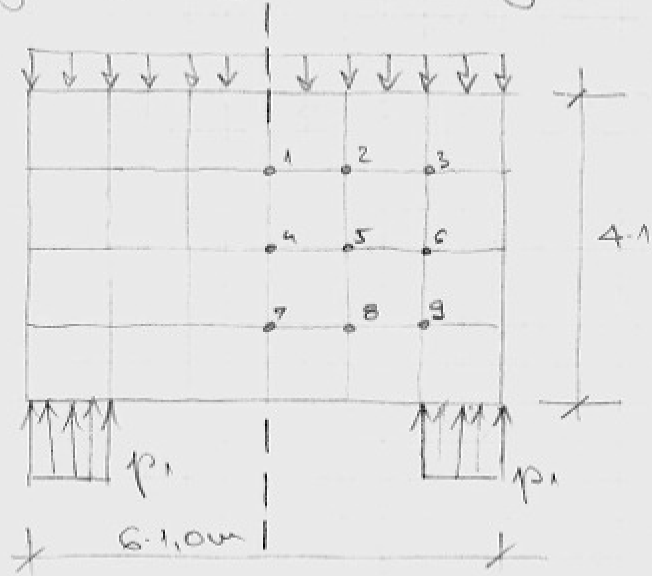
$$F = \int_0^s p_{ny}(x - x_s) ds + \int_0^s p_{nx}(y_s - y) ds = M = M_0 + \overbrace{M_1 \cdot x_1 + M_2 x_2 + M_3 x_3}^{0}$$

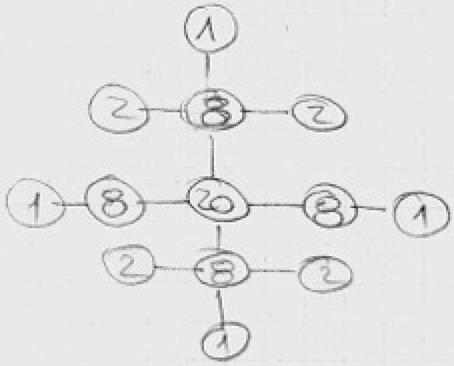
F представља моменат који прави оптерећење

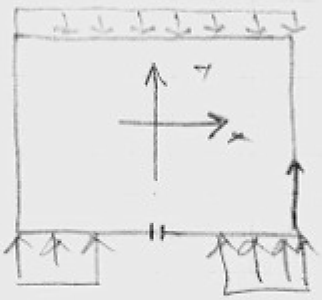
$$\left.\begin{array}{l}\dfrac{\partial F}{\partial x}\\[2ex]\dfrac{\partial F}{\partial y}\end{array}\right\} \rightarrow \frac{\partial F}{\partial n} = \frac{\partial F}{\partial x}\cdot\underbrace{\frac{\partial x}{\partial n}}_{\cos\angle(x,n)} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial n}$$

$F = M_0$

① За дату плочу оптерећену према слици наћи вриједности напонске ф-је и силе у пресјеку у означеним тачкама примјењујући диференцни поступак.

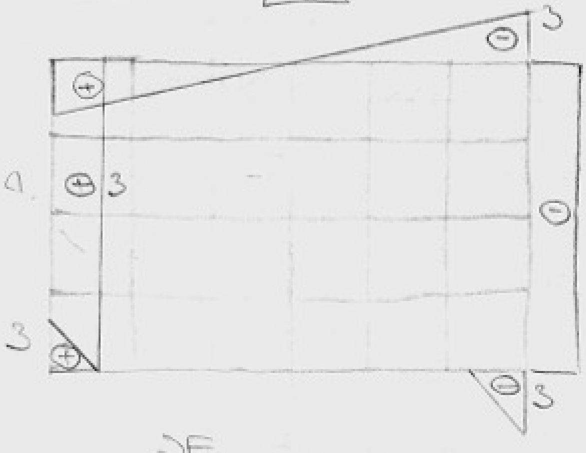






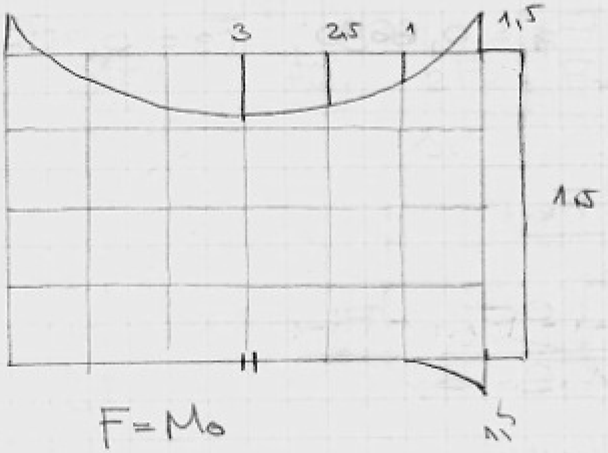
$\frac{\partial F}{\partial y} = Q_x \equiv 0$

$\frac{\partial F}{\partial x} = -Q_y$



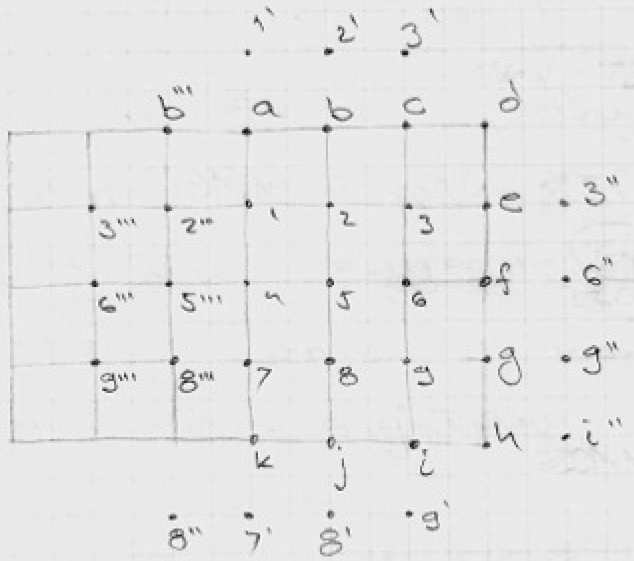
$\frac{\partial F}{\partial x} = -Q_y$

$\frac{\partial F}{\partial n}$



$F = M_0$

Од слободног краја се креће у смјеру супротном од смјера казаљке на сату.



$F_2''' = F_2$

$F_3''' = F_3$

$\vdots$

$F_9''' = F_9$

$$\left(\frac{\partial F}{\partial n}\right)_a = \frac{F_1' - F_1}{2S} = 0 \Rightarrow F_1' = F_1$$

$$F_2' = F_2$$

$$F_3' = F_3$$

$$\left(\frac{\partial F}{\partial n}\right)_e = \frac{F_3'' - F_3}{2S} = -3 \Rightarrow F_3'' = -6 + F_3$$

$F_6'' = -6 + F_6$ $\quad F_9'' = -6 + F_9$ $\quad \left(\frac{\partial F}{\partial n}\right)_k = \frac{F_7' - F_7}{2S} = 0$

$F_7' = F_7$ $\quad F_8' = F_8$ $\quad F_9' = F_9$ $\quad F_8'' = F_8' = F_8$

$k = 1$:

$$20F_1 - 8\cdot(F_a + F_4 + F_2 + F_2''') + 2\cdot(F_b + F_b''' + F_5 + F_5''') + F_1' + F_7 + F_3 + F_3''' = 0$$

$$21F_1 - 16\cdot F_2 + 2F_3 - 8F_4 + 4\cdot F_5 + F_7 = 14$$

$$-8F_1 + 22F_2 - 8F_3 + 2F_4 - 8F_5 + 2F_6 + F_8 = 13.5$$

$k = 9$: $F_3 + 2\cdot F_5 - 8F_6 + 11\cdot F_7 - 8\cdot F_8 + 22\cdot F_9 = 0$

$F_1 = 2.5975$; $F_2 = 2.1855$; $F_3 = 0.8845$; $F_4 = 1.7715$; $F_5 = 1.5$

$F_6 = 0,6078 \quad F_7 = 0,7468;\ F_8 = 0,6680;\ F_9 = 0,2516$

$$(N_x)_k = \left(\frac{\partial^2 F}{\partial y^2}\right)_k \approx \frac{F_l - F_k \cdot 2 + F_i}{S_y^2}$$

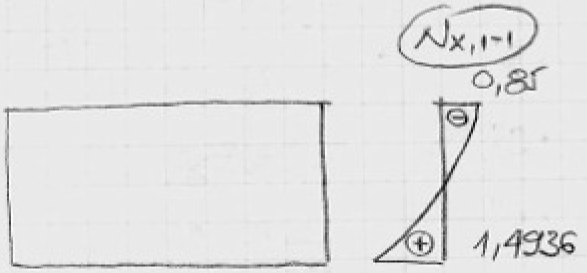
$$(N_y)_k = \left(\frac{\partial^2 F}{\partial x^2}\right)_k \approx \frac{F_{k+1} - 2F_k + F_{k-1}}{S_x^2}$$

$$(N_{xy})_k = -\left(\frac{\partial^2 F}{\partial x \partial y}\right)_k \approx \frac{F_{l-1} + F_{i+1} - F_{l+1} - F_{i-1}}{4 \cdot S_x \cdot S_y}$$

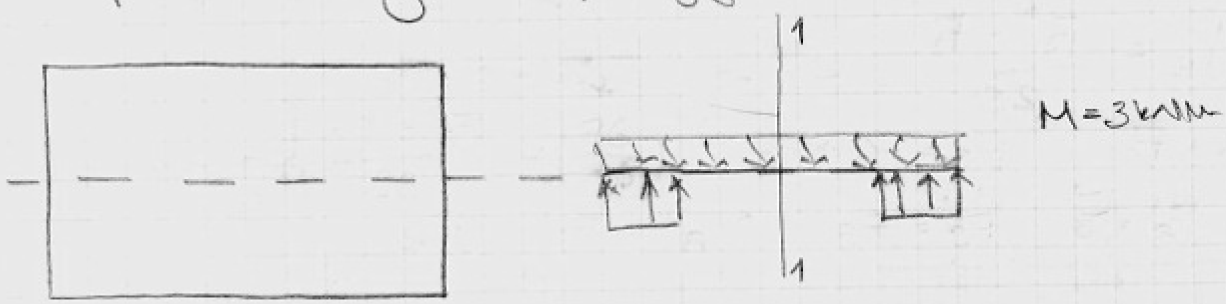
$$N_{xa} \approx \frac{F_1' - 2F_a + F_1}{2 \cdot 1} = -0,85 \text{ kN/m}$$

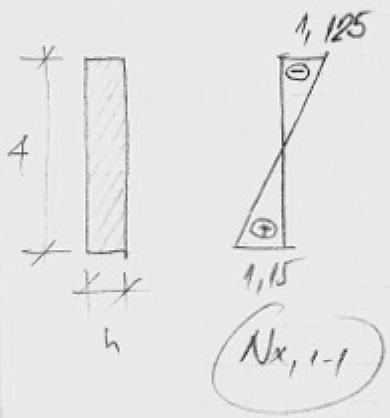
$$N_{x1} \approx \frac{F_a - 2F_a + F_4}{2 \cdot 1} = -0,4235 \text{ kN/m}$$

$N_{x,4} = -0,1987 \quad N_{x,7} = 0,2794 \quad N_{x,12} = 1,4936$ (")



Контрола: $\Sigma X = 0$: (нумерички се израчунају површине позитивног и негативног дијела дијаграма $N_{x,1-1}$ и оне треба да буду једнаке јер нема оптерећења у x правцу)

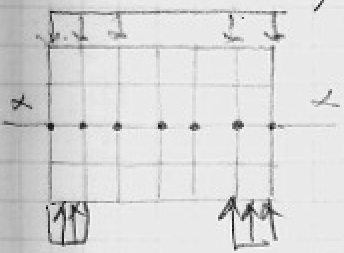


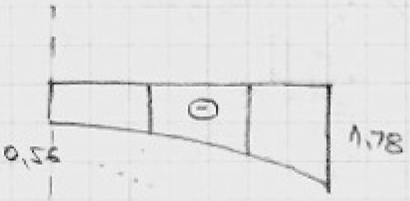


$$\sigma_x = \frac{M}{W} = \frac{M}{\frac{1}{6} \cdot 4^2 h} = \frac{1,125}{h} \quad N_x = 1,125 \text{ kN/m}$$

3.12.'05

$$\left(N_{y_k} = \frac{\partial^2 F}{\partial x^2}\right)_k \approx \frac{F_{k+1} - 2F_k + F_{k-1}}{\Delta x^2}$$





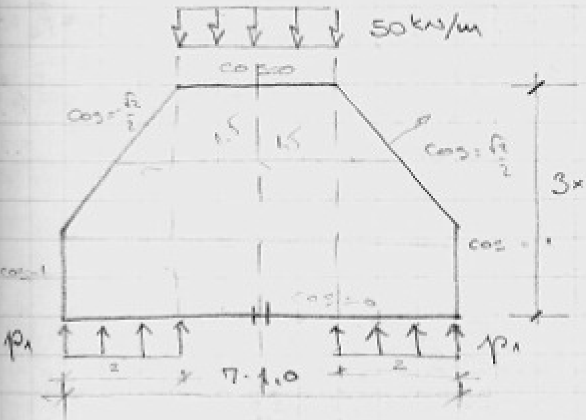
N_y

$$N_{xy,k} = \left(\frac{\partial^2 F}{\partial x \partial y}\right)_k$$

За плочу напрегнуту у својој равни примјеном диференцног поступка:

а) одредити вриједности напонске ф-је и не ног извода по и на контури

b) Нацртати дијаграм N_x у пресјеку L-L

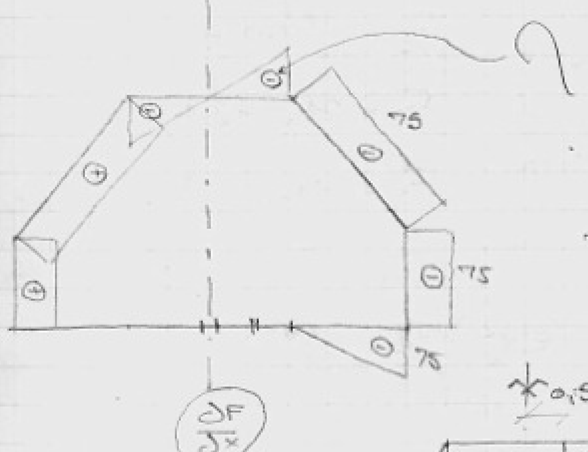


$50 \cdot 3 = 2 \cdot p_1 \cdot 2 \qquad p_1 = 37{,}5 \text{ kN/m}$

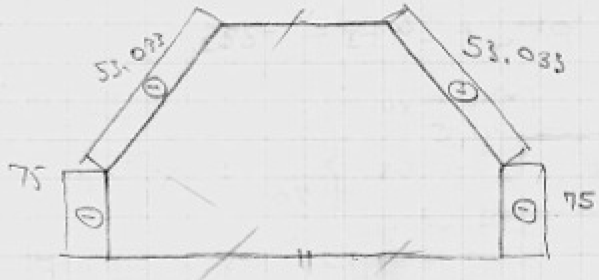
$F = M_0$

$$\frac{\partial F}{\partial n} = \frac{\partial F}{\partial x}\frac{\partial x}{\partial n} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial n} \qquad \frac{\partial F}{\partial x} = -Q_y$$

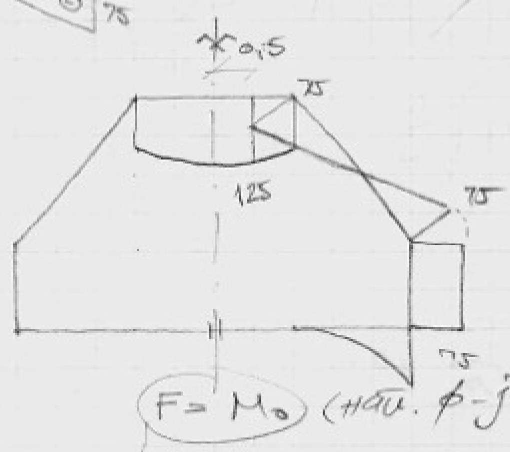
$$\frac{\partial F}{\partial y} = +Q_x \equiv 0$$



$\frac{\partial F}{\partial x}$

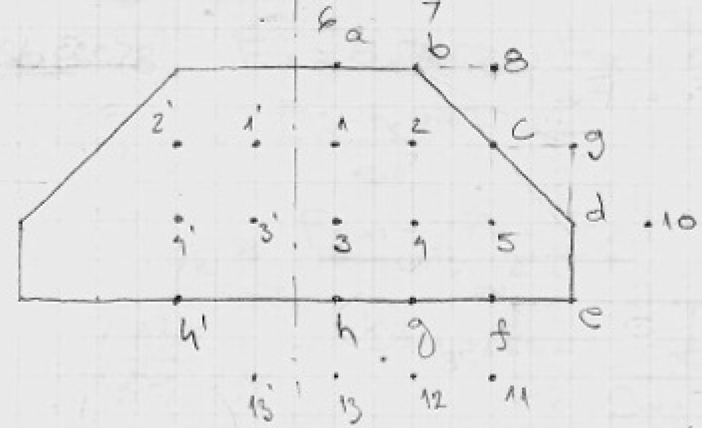


$\frac{\partial F}{\partial n}$



$F = M_0$ (нап. ф-ја)

b)



$$\left(\frac{\partial F}{\partial u}\right)_a = \frac{F_6 - F_1}{2\cdot 1} = 0 \Rightarrow F_6 = F_1 \qquad \left(\frac{\partial F}{\partial u}\right)_d = \frac{F_{10} - F_5}{2\cdot 1} = 75 \Rightarrow$$

$$\Rightarrow F_{10} = -150 + F_5 \qquad \left(\frac{\partial F}{\partial u}\right)_f = \frac{F_{11} - F_5}{2\cdot 1} = 0 \Rightarrow F_{11} = F_5 \; ; \; F_{12} = F_4 \; ;$$

$$F_{13} = F_3$$

$$\left(\frac{\partial F}{\partial u}\right)_1 = \frac{F_8 - F_2}{2\cdot\frac{\sqrt{2}}{2}} = -53{,}033 \qquad \boxed{F_8 = -75 + F_2} \qquad \boxed{F_9 = -75 + F_5}$$

k=1:

$$20F_1 - 8(125 + F_3 + F_{12} + F_1) + 2(75 + 125 + F_4 + F_3) + F_1 + 0 + 0 + F_2 = 0$$

$$\boxed{13F_1 - 7F_2 - 6F_3 + 2F_4 = 600}$$

k=2:

$$\left.\begin{aligned} -7F_1 + 23F_2 - 8F_4 + 3\cdot F_5 &= 575 \\ -6F_1 + 2F_2 + 13F_3 - 7F_4 + F_5 &= -125 \\ 2F_1 - 8F_2 - 7F_3 + 21F_4 - 8F_5 &= 37{,}5 \\ 3F_2 + F_3 - 8F_4 + 24F_5 &= -225 \end{aligned}\right\}$$

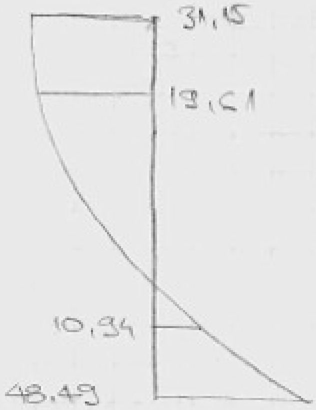
$F_1 = 91{,}6855$

$F_2 = 59{,}4252$

$F_3 = 37{,}4036$

$F_4 = 24{,}2434$

$F_5 = -10{,}2805$



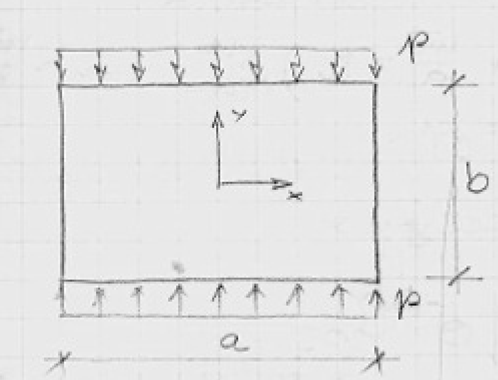
$\frac{a_0}{2}$

$F = Ex^2 + \sum Y_n(u) \cdot$

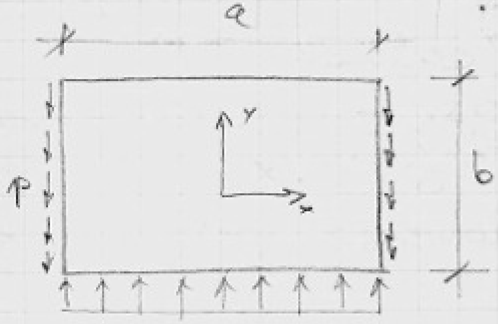
$\frac{a_0}{2}$ $\qquad a_0 = 0$

2. НАПИСАТИ ГР. УСЛОВЕ

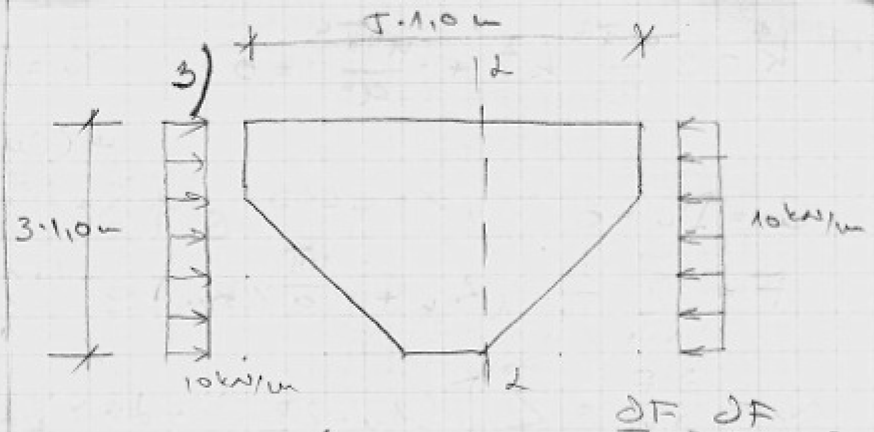
1)



2)



3)

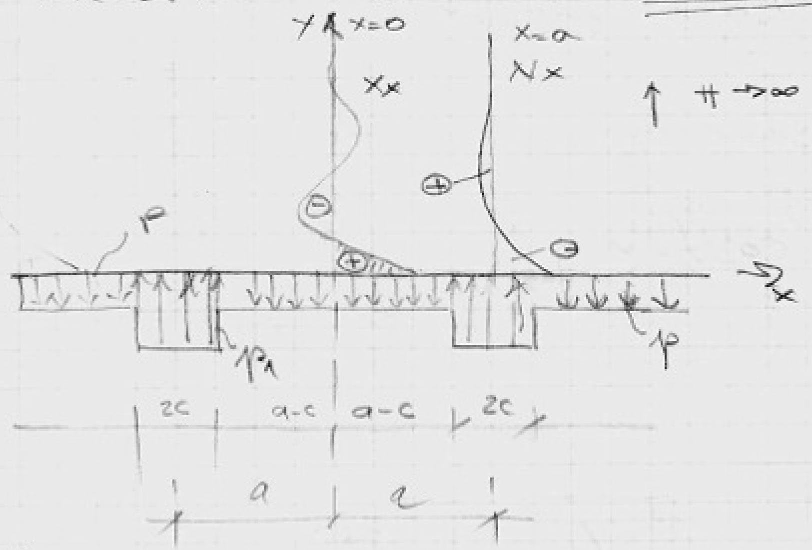


$\frac{\partial F}{\partial y}, \frac{\partial F}{\partial n} = ?$

Применом диференцног поступка одредити силе у пресеку 1-1:

ЗИДНИ НОСАЧИ 6.12.'05.

1. ПЕРИОДИЧНО ОПТЕРЕЋЕЊЕ ПОЛУРАВНИ.



$L = 2a$

$p_1 \cdot 2c = p \cdot 2(a-c)$

$p_1 = p \cdot \frac{a-c}{c}$

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0$$

$$y = 0 \quad \begin{cases} N_y = p(x) & 1) \\ N_{xy} = \emptyset & 2) \end{cases}$$

$$p(x) = \sum_{n=1}^{\infty} a_n \cos \frac{2n\pi}{2a} x$$

$$a_n = \frac{2}{L} \int_0^L p(x) \cdot \cos \frac{n\pi}{a} x \, dx \quad ili \quad \frac{4}{L} \int_0^{L/2} p(x) \cos \frac{n\pi}{a} x \, dx =$$

$$= \frac{p \cdot 2 \cdot a}{\pi c} \, \frac{(-1)^n}{n} \sin \frac{n\pi}{a} \cdot c$$

$$F = \sum_{n=1}^{\infty} Y_n(y) \cos \frac{n\pi}{a} x \qquad Y_n^{IV} - 2 \cdot \frac{n^2\pi^2}{a^2} Y_n'' + \frac{n^4\pi^4}{a^4} Y_n = 0$$

$$k^4 - 2\frac{n^2\pi^2}{a^2}k^2 + \frac{n^4\pi^4}{a^4} = 0 \qquad Y_n = \left(A_n + \frac{n\pi}{a}y B_n\right)e^{-\alpha_n y} +$$

$$+\left(C_n + \frac{n\pi}{a}y D_n\right)e^{\alpha_n y} \qquad \alpha_n = \frac{n\pi}{a}$$

$$C_n = D_n = 0$$

$$F = \sum_{n=1}^{\infty}\frac{1}{\alpha_n^2}\left(A_n + \frac{n\pi}{a}y B_n\right)e^{-\alpha_n y}\cdot\cos\alpha_n x$$

$$N_x = \frac{\partial^2 F}{\partial y^2} = \sum_{n=1}^{\infty}\left[(A_n - 2B_n) + \alpha_n y B_n\right]e^{-\alpha_n y}\cos\alpha_n x$$

$$N_y = \frac{\partial^2 F}{\partial x^2} = \sum_{n=1}^{\infty} -\left(A_n + \alpha_n y B_n\right)e^{-\alpha_n y}\cos\alpha_n x$$

$$N_{xy} = \frac{\partial^2 F}{\partial x \partial y} = -\sum_{n=1}^{\infty}\left(A_n - B_n + \alpha_n y\cdot B_n\right)e^{-\alpha_n y}\sin\alpha_n x$$

1) $-(A_n + 0\cdot B_n) = a_n \qquad A_n = -a_n$

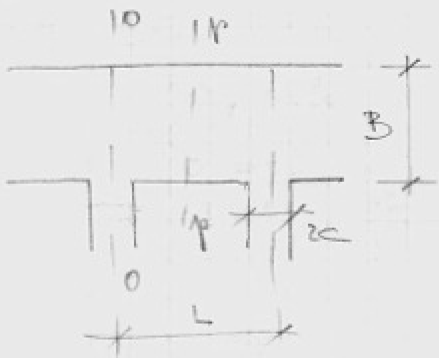
2) $A_n - B_n = 0 \qquad \boxed{B_n = A_n = -a_n}$

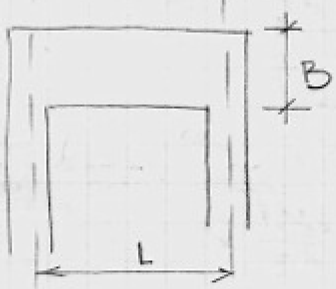
$$F = -\frac{a^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}a_n(1+\alpha_n y)e^{-\alpha_n y}\cos\alpha_n x$$

$$N_x = \sum_{n=1}^{\infty}a_n(1-\alpha_n y)e^{-\alpha_n y}\cos\alpha_n x$$

$$N_y = \sum_{n=1}^{\infty}a_n(1+\alpha_n y)\cdot e^{-\alpha_n y}\cos\alpha_n x$$

$$N_{xy} = \sum_{n=1}^{\infty}a_n\alpha_n y e^{-\alpha_n y}\sin\alpha_n x$$



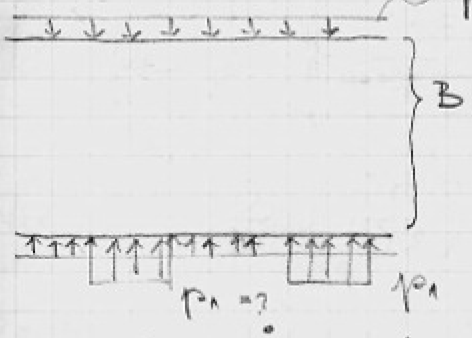


$$F = \sum_{n=1}^{\infty}Y_n(y)\cdot\cos\alpha_n x$$

$$''(A_n + \alpha_n y\cdot B_n)e^{-\alpha_n y} + (C_n + \alpha_n y D_n)e^{\alpha_n y}$$

$Y=0$: N_y, N_{xy}

$Y=b$: N_y, N_{xy}



$p_g = \frac{a_0}{2} + \sum p_{ng} \cdot \cos \alpha_n x$ ✓

$p_d = \frac{a_0}{2} + \sum p_{nd} \cos \alpha_n \cdot x$

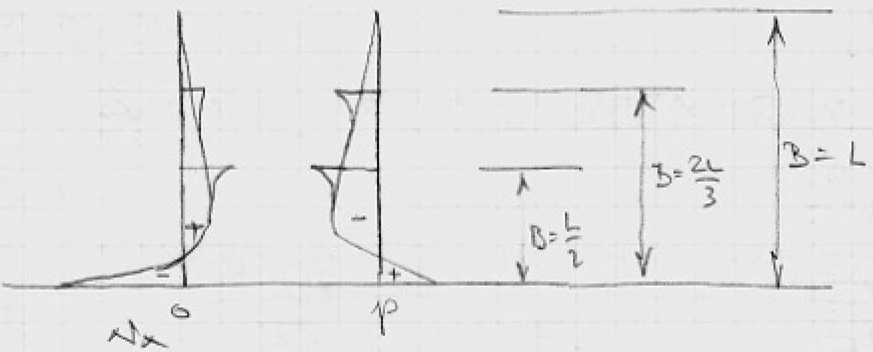
$N_x = f(B/L)$, $f(2c)$ $\boxed{B/L < 0{,}45}$ — прибл. греда $N_{x\,p-p}$

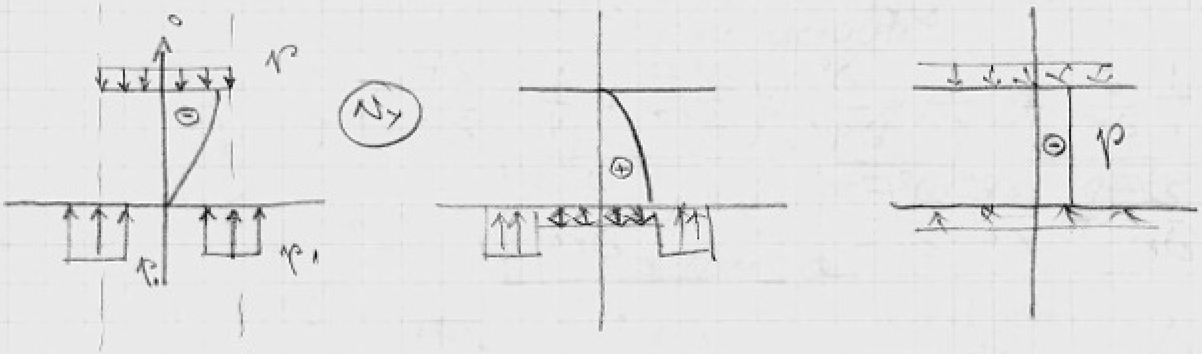
$N_{x\,0-0} = f(2c)$ - зависи од ширине ослањања!

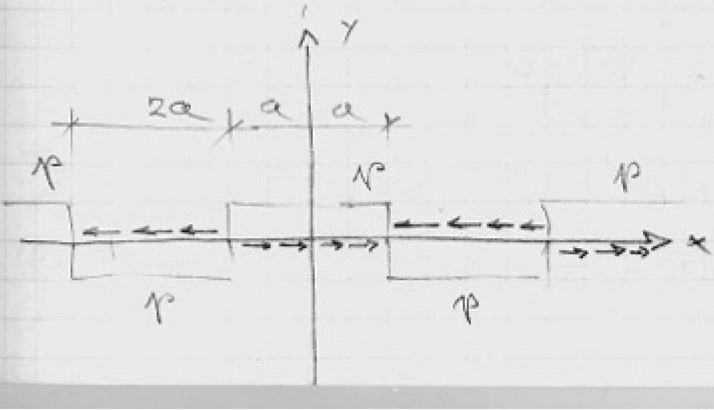
$\frac{c}{L}$

$\frac{B}{L} > 1 \Rightarrow$ зид се сматра као полураван!

$1 > \frac{B}{L} > 0{,}45 \Rightarrow$ нумерички! (зидни носач - плоча н.у.с.р)



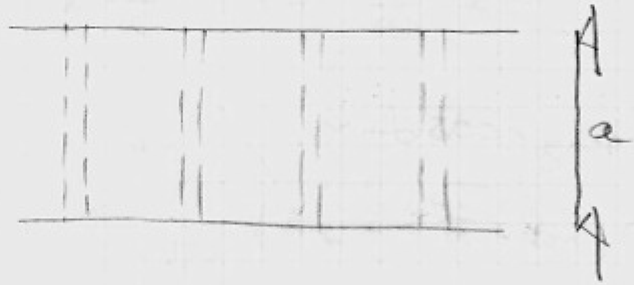


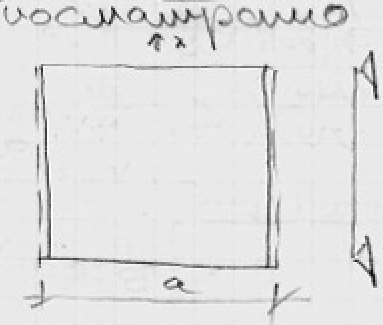


$L = 4a$ $p(x) = \sum p_n \cos \frac{2n\pi x}{4a}$

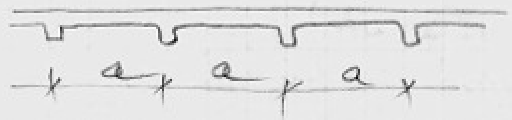
$Y=0$: $\begin{cases} Y = 0 \\ N_{xy} = -p = \sum p_n \cos \end{cases}$

посматрамо једно пол[illegible]

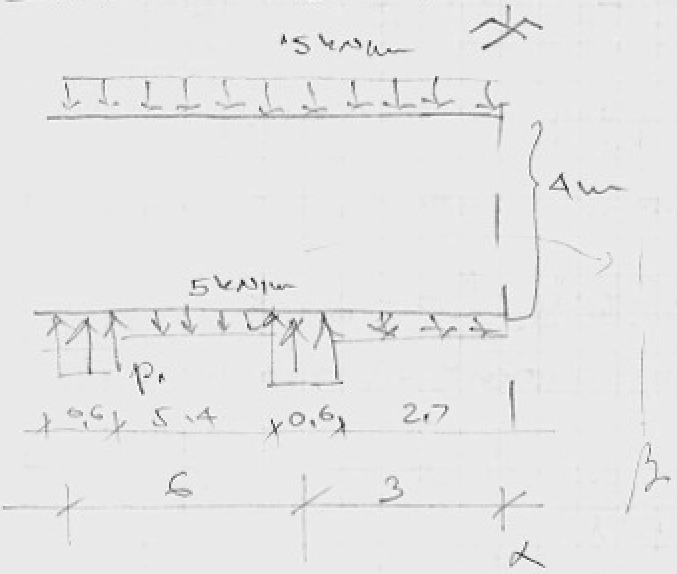




$y = \pm \frac{a}{2}$ $\quad w^{pl} = w^{G}$ $\quad \frac{\partial w^{pl}}{\partial y} = 0$



~~Домаћи рад~~:



$F = \frac{p_0}{2} x^2$

$n = 2$

a_0, a_1

13. 12. 2005.

РАВНО НАПРЕЗАЊЕ У ПОЛАРНИМ КООРДИНАТАМА:

r, φ

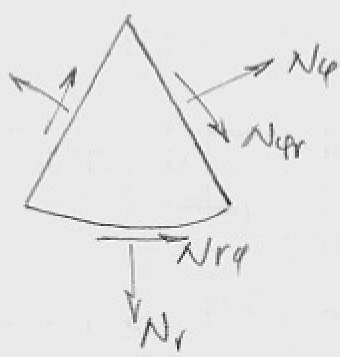
$$\frac{\partial^4 F}{\partial x^4} + 2 \cdot \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0$$

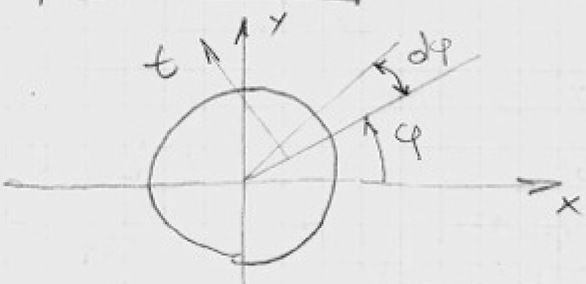
правоугле координате!

$$N_x = \frac{\partial^2 F}{\partial y^2} \qquad N_y = \frac{\partial^2 F}{\partial x^2} \qquad N_{xy} = -\frac{\partial^2 F}{\partial x \partial y}$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2} \cdot \frac{\partial^2}{\partial \varphi^2}\right)\left(\frac{\partial^2 F}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial \varphi^2}\right) = 0$$

$$\boxed{N_r = \frac{1}{r}\frac{\partial F}{\partial r} + \frac{1}{r^2}\frac{\partial^2 F}{\partial r^2}} \qquad \boxed{N_\varphi = \frac{\partial^2 F}{\partial r^2}} \qquad N_{r\varphi} =$$





$N_r = \frac{\partial^2 F}{\partial t^2}$ $\quad dt = r \cdot d\varphi \quad$ $N_t = \frac{\partial^2 F}{\partial r^2} = N_\varphi$

$N_{rt} = -\frac{\partial^2 F}{\partial r \partial t}$ $\quad N_x + N_y = \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial x^2} = \Delta F$

$$N_r + N_t = \Delta F = \frac{\partial^2 F}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial F}{\partial r} + \frac{1}{r^2} \frac{\partial^2 F}{\partial \varphi^2}$$

$" \frac{\partial^2 F}{\partial r^2}$

$$N_{\varphi r} = -\frac{\partial}{\partial r}\left(\frac{\partial F}{r \partial \varphi}\right) = \boxed{\frac{1}{r^2}\frac{\partial F}{\partial \varphi} - \frac{1}{r}\frac{\partial^2 F}{\partial \varphi^2}}$$

$p = p(r)$ ротационо-симетрично оптерећење (не зав. (φ))

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)\left(\frac{d^2 F}{dr^2} + \frac{1}{r}\frac{dF}{dr}\right) = 0$$

$$\boxed{N_r = \frac{1}{r}\frac{dF}{dr} \quad \Big| \quad N_\varphi = \frac{d^2 F}{dr^2} \quad \Big| \quad N_{r\varphi} \equiv 0}$$

$$-\frac{d^4 F}{dr^4} + \frac{2}{r}\frac{d^3 F}{dr^3} - \frac{1}{r^2}\frac{d^2 F}{dr^2} + \frac{1}{r^3}\frac{dF}{dr} = 0$$

Ојлерова диф. једначина:

смјена: $r = e^t$

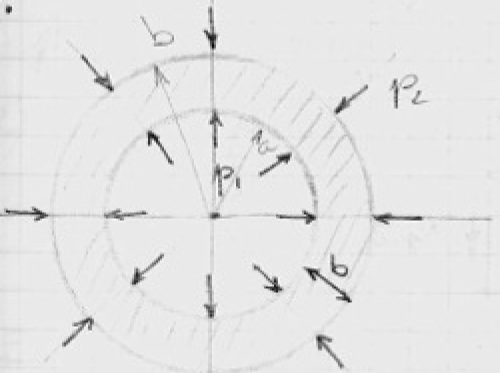
$$F = D + A \cdot \ln r + B r^2 + C \cdot r^2 \ln r$$

$N_r = \frac{A}{r^2} + 2B + C(1 + 2 \cdot \ln r)$ $\quad N_\varphi = -\frac{A}{r^2} + 2B + C(3 + 2\ln r)$

$N_{r\varphi} = 0$

D не утиче на напрезање па се може ставити да је $= 0$

1.



гранични услови:

$r = a: \begin{cases} N_r = -p_1 \\ N_\varphi \text{ - не може да се јави!} \\ N_{r\varphi} \equiv 0 \end{cases}$

$r = b: \begin{cases} N_r = -p_2 \quad (2) \end{cases}$

Треба нам још једна једначина. Посматрамо деформацију плоче $\varepsilon_r = \frac{du}{dr}$ $\quad \varepsilon_\varphi = \frac{u}{r} \Rightarrow u = r \cdot \varepsilon_\varphi \quad \gamma_{r\varphi} \equiv 0$

$$\frac{du}{dr} = \varepsilon_\varphi + r \cdot \frac{d\varepsilon_\varphi}{dr} \qquad \boxed{\varepsilon_r = \varepsilon_\varphi + r \cdot \frac{d\varepsilon_\varphi}{dr}}$$

$$\varepsilon_r = \frac{1}{Eh}\left(N_r - \nu \cdot N_\varphi\right) \qquad \varepsilon_\varphi = \frac{1}{Eh}\left(N_\varphi - \nu \cdot N_r\right)$$

$$2 \cdot (1+\nu)\left(c - \frac{A}{\nu r^2}\right) + 2r\left(\frac{A}{r^3} + \frac{c}{r}\right) + \nu\left(\frac{A}{r^3} - \frac{c}{r}\right) = 0$$

$$\boxed{c = 0}$$

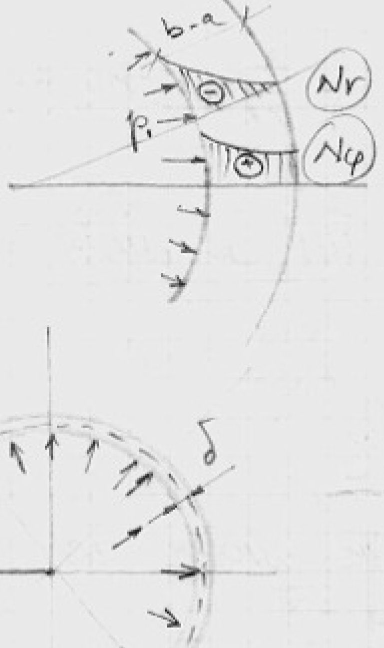
$$\left.\begin{aligned} \frac{A}{a^2} + 2B &= -p_1 \\ \frac{A}{b^2} + 2B &= -p_2 \end{aligned}\right\} \qquad A = -\frac{p_1 - p_2}{b^2 - a^2} \cdot a^2 b^2 \qquad B = -\frac{p_2 b - p_1 \cdot a^2}{2(b^2 - a^2)}$$

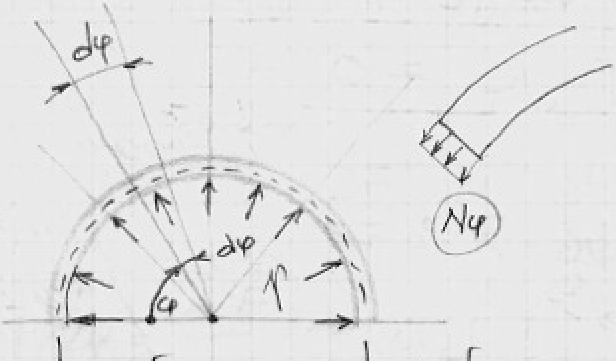
$$N_r = \frac{-p_2 b^2 - p_1 a^2}{b^2 - a^2} - \frac{p_1 - p_2}{b^2 - a^2}\,\frac{a^2 b^2}{r^2}$$

$$N_\varphi = -\frac{p_2 b^2 - p_1 a^2}{b^2 - a^2} + \frac{p_1 - p_2}{b^2 - a^2} \cdot \frac{a^2 b^2}{r^2}$$

$\underline{p_2 = 0}$

$$N_r = \frac{p_1 a^2}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right) \qquad N_\varphi = \frac{p_1 a^2}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$





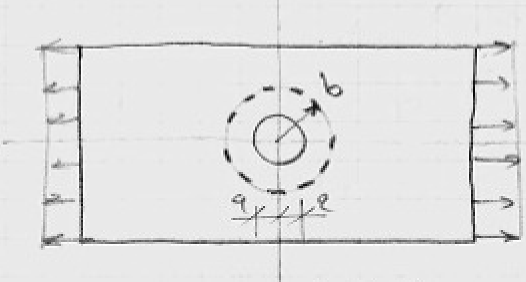
$$r = \frac{a+b}{2} \qquad \Sigma V = 0: \; 2 \cdot N_\varphi \cdot \delta = \int_0^\pi p \cdot \sin\varphi \cdot r \cdot d\varphi \qquad \boxed{N_\varphi = \frac{p \cdot r}{\delta}}$$

$N_\varphi = ?$

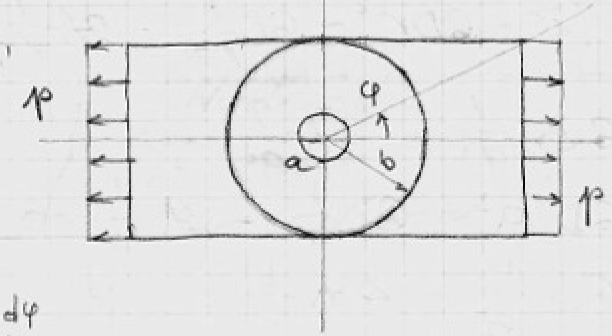
КОТЛОВСКА
Ф-ЛА!

Посматрамо пола плоче а остатак смо замијенили реакцијама. Претпоставили смо равномјерну расподјелу силе $N\varphi$!

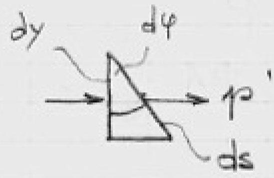
2. Плоча аксијално оптерећена и ослабљена отвором $2a$ који је много мањи од висине и ширине плоче. Да нема отвора плоча би била аксијално напрегнута и јавила би се сила $N_x = p$.
У околини отвора се јавља поремећено напонско стање које је локалног карактера. У околини отвора $N_x \neq p$, а са удаљавањем од отвора је $N_x = p$



$b \gg a$



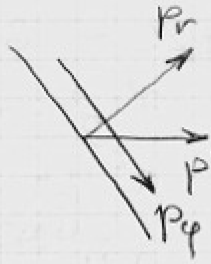
$\underline{N_x = p}$



$p \cdot dy = p' ds$

$p' = p \cdot \frac{dy}{ds}$

$\boxed{p' = p \cdot \cos\varphi}$



$$p_r = p' \cdot \cos\varphi = p \cdot \cos^2\varphi = \frac{p}{2}(1 + \cos 2\varphi)$$

$$p_\varphi = -p' \cdot \sin\varphi = -p \cdot \sin\varphi \cdot \cos\varphi = -\frac{p}{2}\sin 2\varphi$$

a) $p_{r_1} = \frac{p}{2}$

b) $\boxed{\begin{aligned} p_{r_2} &= \frac{p}{2} \cdot \cos 2\varphi \\ p_\varphi &= -\frac{p}{2}\sin 2\varphi \end{aligned}}$

$F(r, \varphi) = f(r) \cos 2\varphi$

$r = a: \begin{cases} N_r = 0 \\ N_{r\varphi} = 0 \end{cases}$ $\quad r = b: \begin{cases} N_r = p_{r_2} = \frac{p}{2}\cos 2\varphi \\ N_\varphi = p_\varphi = -\frac{p}{2} \cdot \sin 2\varphi \end{cases}$

$\Delta\Delta F = 0 \quad / : \cos 2\varphi$

$$\frac{d^4 f}{dr^4} + \frac{2}{r}\cdot\frac{d^3 f}{dr^3} - \frac{9}{r^2}\frac{d^2 f}{dr^2} + \frac{9}{r^3}\frac{df}{dr} = 0$$ Ојлерова диф. једначин.

$r = e^t$

$$\frac{d^4 f}{dt^4} - 4\cdot\frac{d^3 f}{dt^3} - 4\frac{d^2 f}{dt^2} + 16\cdot\frac{df}{dt} = 0$$

$$F = \left(A + B\frac{1}{r^2} + C\cdot r^2 + D\cdot r^4\right)\cdot\cos 2\varphi$$

$$N_r = -\left(2C + 6B\frac{1}{r^4} + 4\cdot A\cdot\frac{1}{r^2}\right)\cdot\cos 2\varphi$$

$$N_\varphi = \left(2C + 12Dr^2 + 6B\frac{1}{r^4}\right)\cos 2\varphi$$

$$N_{r\varphi} = \left(2C + 6Dr^2 - 6B\frac{1}{r^4} - 2A\frac{1}{r^2}\right)\sin 2\varphi$$

$$2C + 6\cdot B\cdot\frac{1}{a^4} + 4A\frac{1}{a^2} = 0$$

$$2C + 6D\cdot a^2 - 6B\frac{1}{a^4} - 2A\frac{1}{a^2} = 0$$

$$2C + 6B\frac{1}{b^4} + 4A\frac{1}{b^2} = +p/2$$

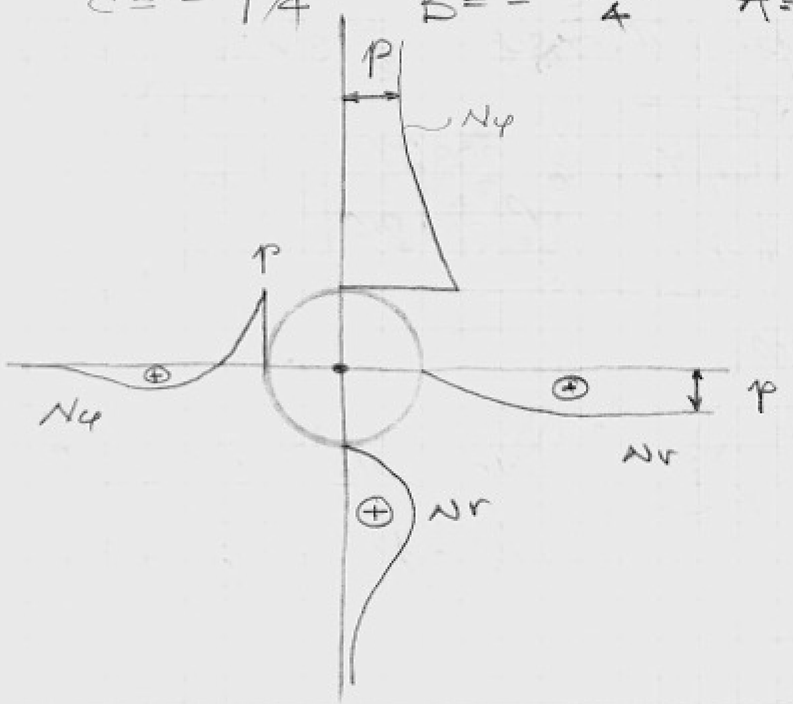
$$2C + 6D\cdot b^2 - 6B\cdot\frac{1}{b^4} - 2A\frac{1}{b^2} = -p/2$$

$a^2/b^2 \to 0$

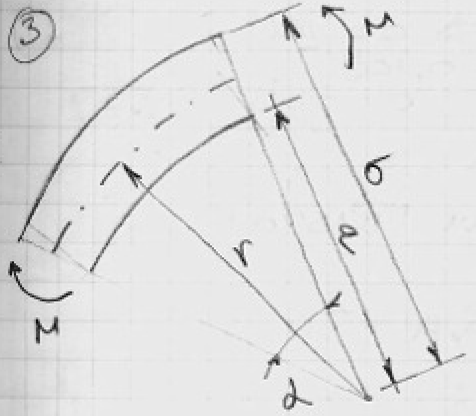
$a^4/b^4 \to 0$

$a/b \to 0$

$D = 0 \qquad C = -p/4 \qquad B = -\frac{p\cdot a^4}{4} \qquad A = \frac{pa^2}{2}$



③



$$F = D + A \cdot \ln r + B r^2 + C \cdot r^2 \cdot \ln r$$

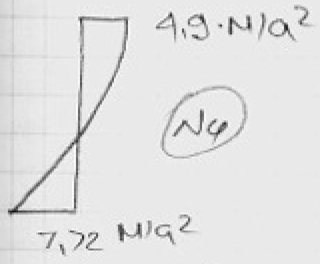
$$N_r = \frac{1}{r} \cdot dF/dr \quad N_\varphi = d^2F/dr^2 \quad N_\varphi \equiv 0$$

Носач у принципу није ротационо симетрично оптерећен али се моменти могу представити одговарајућим нормалним силама. Сваки пресјек ће бити исто напрегнут па се може примјенити принцип ротационе симетрије!

Гранични услови:

$r=a$: $\{ N_r = 0 \qquad \varphi = 0 \qquad \{ \int_a^b N_\varphi dr = 0$

$r=b$: $\{ N_{r\varphi} \equiv 0 \qquad \varphi = \alpha \qquad \{ \int_a^b N_\varphi \cdot r\, dr = -M$



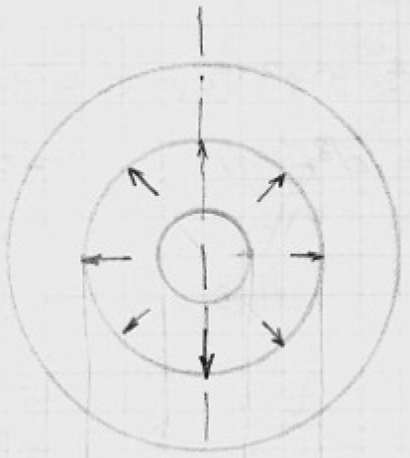
$b/a = 2$

Ако бисмо овај носач третирали као линијски:

$N_{\varphi,max} = \frac{M}{W} = 6M/a^2$

Нисмо на страни сигурности јер је мања сила у доњој зони, а већа у горњој што је неповољно за случај бетонске плоче!

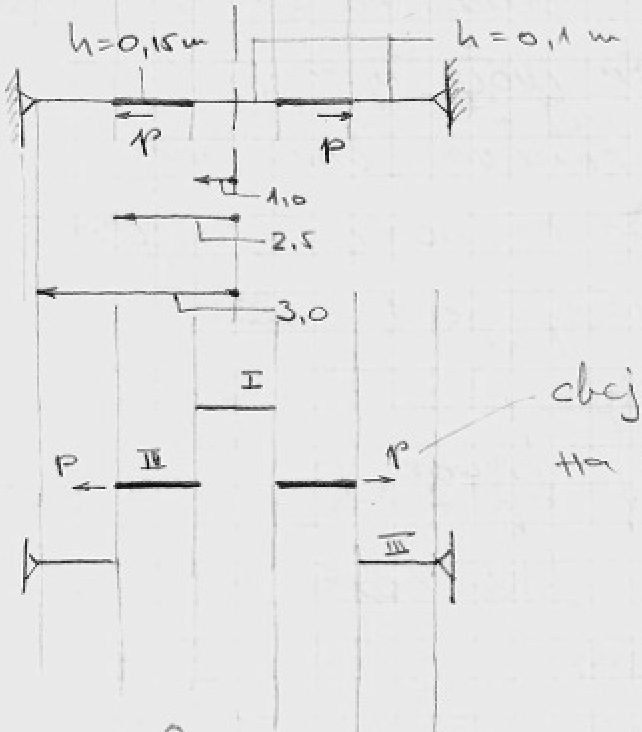
4



$E = 30\ GPa \quad \nu = 0{,}16$

$p = 100\ kN/m$

ротационо сим. плоча!



свеједно је да ли ће се p ставити на блоку II или III

$$F_I = \cancelto{0}{D_1} + A_1 \ln r + B_1 r^2 + C_1 \cdot r^2 \ln r$$

$$F_I = B_1 r^2 \qquad N_{rI} = \frac{1}{r}\frac{dF_I}{dr} = 2B_1$$

$$N_{\varphi I} = \frac{d^2 F_I}{dr^2} = 2B_1$$

$$F_{II} = D_2 + A_2 \ln r + B_2 r^2 + \cancelto{0}{C_2'} \cdot r^2 \ln r$$

$$N_{rII} = \frac{A_2}{r^2} + 2B_2$$

$$N_{\varphi II} = -\frac{A_2}{r^2} + 2B_2$$

$$F_{III} = \cancelto{0}{D_3'} + A_3 \ln r + B_3 r^2 + C_3 \cdot r^2 \ln r$$

$$N_{rIII} = \frac{A_3}{r^2} + 2B_3 \qquad N_{\varphi III} = -\frac{A_3}{r^2} + 2B_3$$

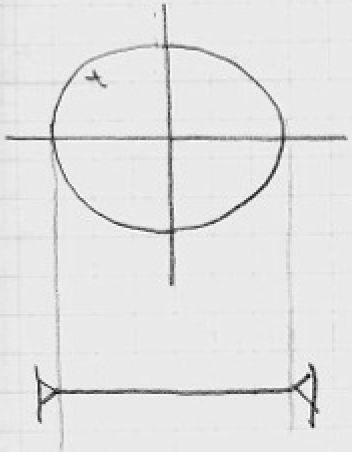
Имамо 5 непознатих константи које ћемо израчунати из 5 услова: граничних и прелазних

$$r = 3m \begin{cases} u_{III} = 0 \Leftrightarrow \varepsilon\varphi_{III}(r=3) = 0 \\ \Downarrow \\ \varepsilon\varphi_{III} = 0 \\ \varepsilon\varphi = \frac{1}{Eh}(N\varphi - \nu N r) \end{cases}$$

$$r = 2{,}5m \begin{cases} N r_{III} + p = N r_{II} \\ u^{II} = u^{III} \end{cases} \quad ?$$

$$r = 1m \begin{cases} N_r^{I} = N_r^{II} \\ u^{I} = u^{II} \end{cases}$$

(5) Плоча оптерећена равномјерном темп. t



$$F = D + A \ln r + B r^2 + C r^2 \ln r$$

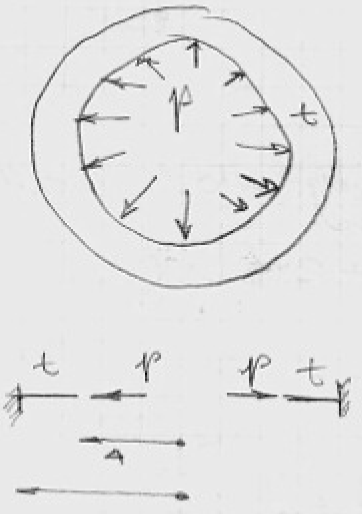
$$r = a \begin{cases} u = 0 \Rightarrow \varepsilon\varphi = 0 \end{cases}$$

$$\varepsilon\varphi = \frac{1}{Eh}(N\varphi - \nu N_r) + \alpha_t \cdot t$$

$$\varepsilon r = \frac{1}{Eh} \cdot (N r - \nu \cdot N\varphi) + \alpha_t \cdot t$$

20.12.'05

1. За дату плочу одредити израз за помјерање и изразе за с.у.и. Плоча је по унутрашњој контури укљештена а по спољашњој слободна!



$h = 10\,cm$

$E = 30\ GPa$

$\nu = 0.2$

$\alpha_t = 10^{-5}\ 1/°C$

$t = -20°C$

$p = 5\ kN/m$

$$F = D + A\ln r + Br^2 + Cr^2 \ln r$$

$r = 4m \quad \left\{ N_r = -p \right.$

$r = 6m \quad \left\{ \begin{array}{l} u = 0 \\ r,\ \varepsilon_\varphi \quad \varepsilon_\varphi = 0 \end{array} \right.$ $\quad 2) \quad N_r = \dfrac{A}{r^2} + 2B$

$N_\varphi = -\dfrac{A}{r^2} + 2B \qquad r = 4. \quad \dfrac{A}{r^2} + 2B = -5 \ /\cdot 16$

$$\boxed{A + 32B = -80}$$

$$\varepsilon_\varphi = \frac{1}{Eh}\left(N_\varphi - \nu N_r\right) + \alpha_t \cdot t =$$

$$= \frac{1}{Eh}\left(-\frac{A}{r^2} + 2B - \nu\cdot\frac{A}{r^2} - 2\nu B\right) + \alpha_t \cdot t =$$

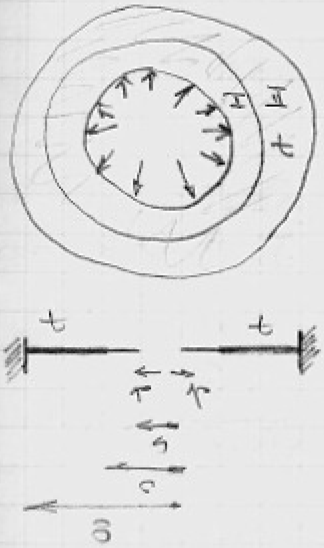
$$= \frac{1}{Eh}\left[(-1-\nu)\frac{A}{r^2} + (1-\nu)\,2B\right] + \alpha_t \cdot t$$

$r = 6 \quad \dfrac{1}{Eh}\left[-1.2\cdot\dfrac{A}{6^2} + 0.8\cdot 2B\right] + 10^{-5}(-20) = 0 \ /Eh$

A,B $u = r \cdot \varepsilon_\varphi$ $\varepsilon = \frac{N}{EF} \cdot \alpha_t \cdot t^\circ$

2. За плочу на сл. одредити изразе за помјерање и силе у пресјеку

стопа



$h_1 = 6\,cm$ $E = 30\,GPa$

$h_2 = 10\,cm$ $\nu = 0{,}2$

$p = 4\,kN/m$ темп. дјелује на

$t = 10^\circ C$ плочи 2 само

$\alpha_t = 10^{-5}$.

$F_I = A_1 \ln r + B_1 r^2$ $F_{II} = A_2 \cdot \ln r + B_2 r^2$

$r = 4\,m \quad \left\{ N_r^{I} = -p \right.$ $r = 6\,m \quad \left\{ \begin{array}{l} u^{I} = u^{II} \\ N_r^{I} = N_r^{II} \end{array} \right.$

$r = 8 \quad u^{II} = 0$

$u^{I} = \frac{1}{Eh_1}\left(N_\varphi^{I} - \nu \cdot N_r^{I}\right)$

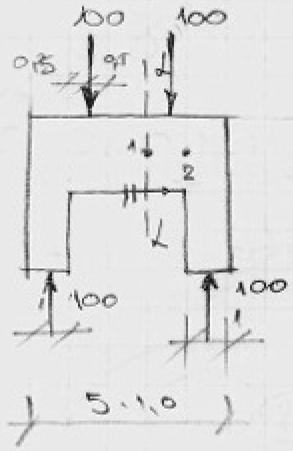
$u^{II} = \frac{1}{Eh_2}\left(N_\varphi^{II} - \nu N_r^{II}\right) + \alpha_t \cdot t$

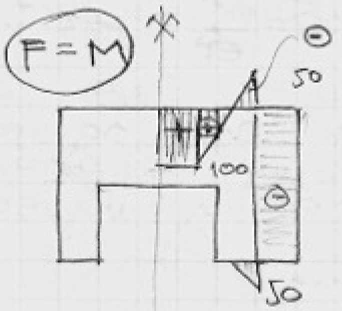
$N_r^{I} = \frac{A_1}{r^2} + 2B_1$ $N_\varphi^{I} = -\frac{A_1}{r^2} + 2B_1$ $N_r^{II} = \frac{A_2}{r^2} + 2B_2$

$N_\varphi^{II} = -\frac{A_2}{r^2} + 2B_2$

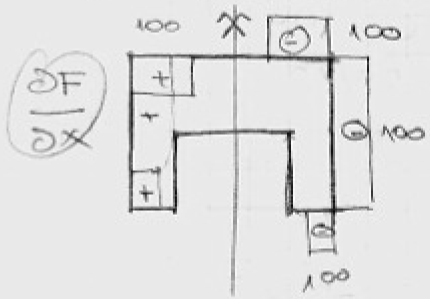
3. За плочу на слици користећи диференцни поступак:

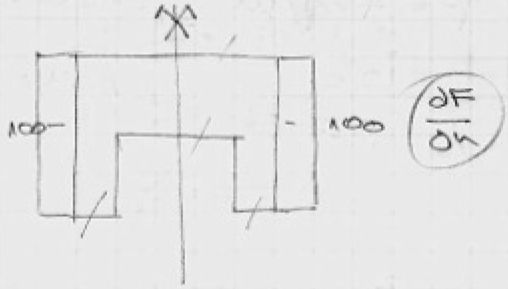
- одредити вриједности напонске ф-је и њеног извода $\frac{dF}{dn}$ на контури!
- срачунати пресјечне силе у означеном пресјеку!



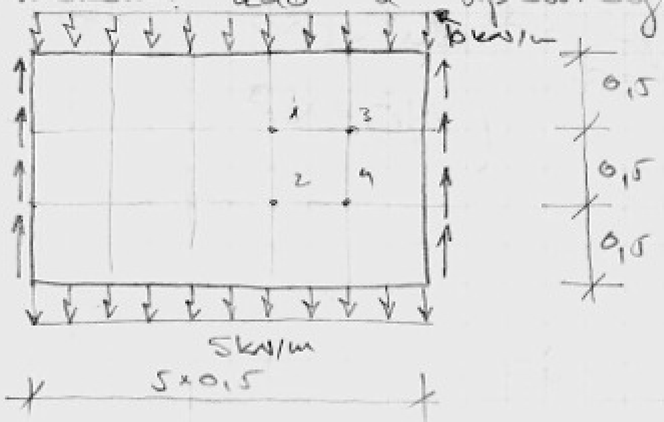


$$\frac{\partial F}{\partial n} = \frac{\partial F}{\partial x}\cdot\frac{\partial x}{\partial n} + \frac{\partial F}{\partial y}\cdot\frac{\partial y}{\partial n} \qquad \frac{\partial F}{\partial x} = -Q_y \qquad \frac{\partial F}{\partial y} = Q_x = 0$$





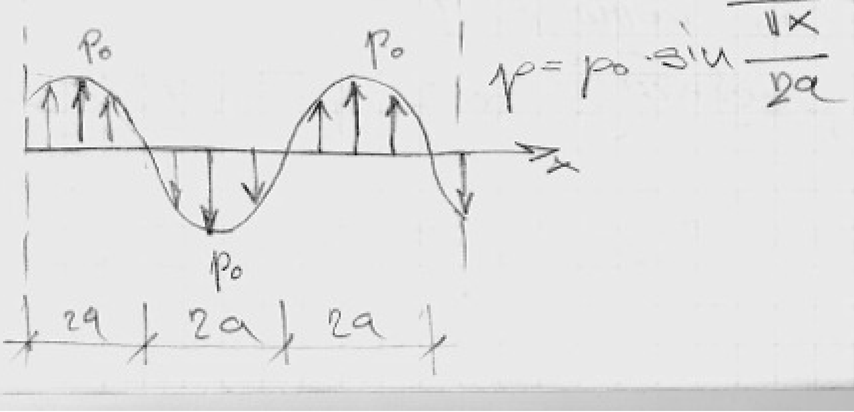
4. текст: као и претходни



N_x, N_{xy} = ? α-α

(полуравни)

5. За 1/2 раван оптерећену периодичним антиметр. оптерећењем одредити израз за силе у пресјеку!



$$p = p_0 \cdot \sin\frac{\pi x}{2a}$$

$$F = Y_1(y)\sin\frac{\pi x}{2a} \qquad y=0 \quad \begin{cases} N_y = p(x) \\ N_{xy} = 0 \end{cases}$$

$$F = \frac{1}{\alpha_1^2}\left(A_1 + \frac{\pi y}{2a}B_1\right)e^{-\frac{\pi y}{2a}}\sin\frac{\pi x}{2a} \quad ; \quad \alpha_1 = \frac{\pi}{2a}$$

...

$$A_1 = -p_0 \qquad B_1 = -p_0$$

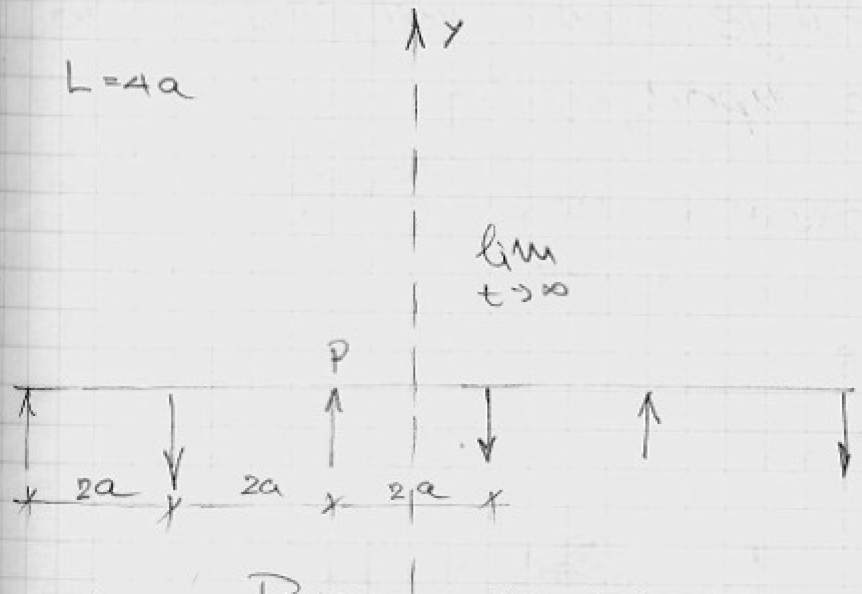
$$N_x = \frac{\partial^2 F}{\partial y^2} = p_0 e^{-\alpha_1 y}(\alpha_1 y - 1)\sin\alpha_1 x$$

$$N_y = \frac{\partial^2 F}{\partial x^2} = -p_0 e^{-\alpha_1 y}(1+\alpha_1 y)\sin\alpha_1 x$$

$$N_{xy} = \frac{\partial^2 F}{\partial x \partial y} = -p \cdot \alpha_1 y e^{-\alpha_1 y}\cos\alpha_1 x$$

6. исти текст:

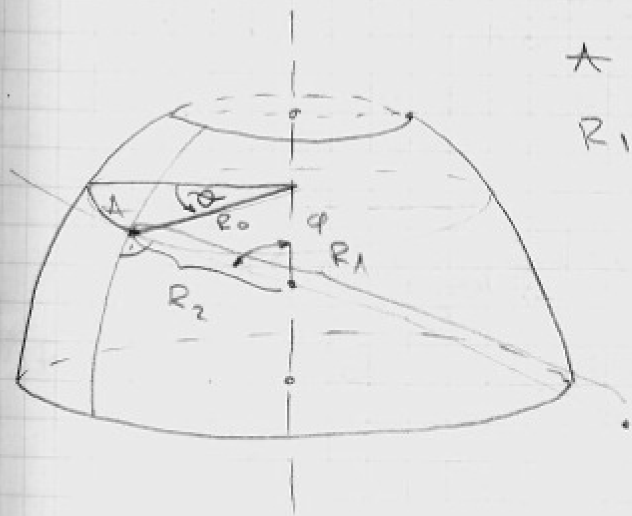
$L = 4a$



развијемо у ред и даље је исто као претходни!

16. 2. 2006.

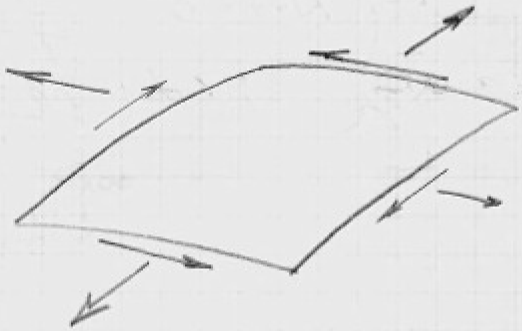
$\frac{h}{R} \ll 1$ **Ротационе љуске**



$A(\varphi, \theta)$

R_1, R_2

МЕМБРАНСКА ТЕОРИЈА РОТАЦИОНИХ ЉУСКИ



$$K = \frac{Eh^3}{12(1-\nu^2)}$$

$1^\circ \quad \frac{h}{R} \ll 1$

$N_{\alpha\beta} = N_{\beta\alpha}$

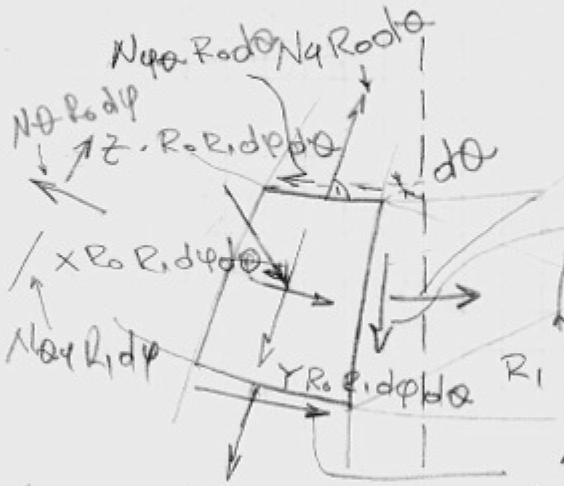
$N_{\varphi\theta} = N_{\theta\varphi}$

2° нема наглих промјена дебљине љуске!

3° оптерећење: не смије бити концентрисано и мора бити благо промјењиво

4° на мјесту ослањања мора бити мембранско стање напона

$$\left.\begin{matrix} N_\varphi, N_\theta \\ N_{\varphi\theta} = N_{\theta\varphi} \end{matrix}\right\}$$



$$\left(N_\theta + \frac{\partial N_\theta}{\partial \theta} d\theta\right) R_1 d\varphi$$

$$\left(N_{\theta\varphi} + \frac{\partial N_{\theta\varphi}}{\partial \theta} d\theta\right) R_1 \cdot d\varphi$$

$$N_{\varphi\theta} R_0 + \frac{\partial}{\partial \varphi}\left(N_{\varphi\theta} R_0\right) d\varphi \,\Big)\, d\theta$$

$$\left(N_\varphi R_0 + \frac{\partial}{\partial \varphi}\left(N_\varphi \cdot R_0\right) d\varphi\right) d\theta$$

1) $\frac{\partial N_\theta}{\partial \theta} R_1 + \frac{\partial}{\partial \varphi}(N_{\varphi\theta} \cdot R_0) + N_{\varphi\theta} \cdot R_1 \cdot \cos\varphi + X R_0 R_1 = 0$

2) $\frac{\partial}{\partial \varphi}(N_\varphi \cdot R_0) + \frac{\partial N_{\varphi\theta}}{\partial \theta} \cdot R_1 - N_\theta \cdot R_1 \cdot \cos\varphi + Y R_0 \cdot R_1 = 0$

3) $\frac{N_\varphi}{R_1} + \frac{N_\theta}{R_2} + Z = 0$

$N_\varphi, N_\theta, N_{\varphi\theta}$

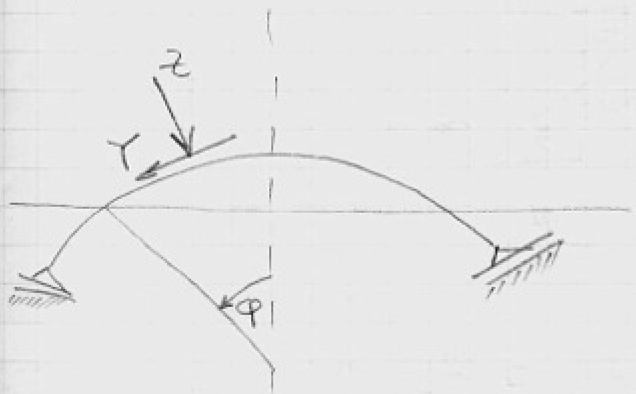
Ако је ротациона површ оптерећена ротационо симетричним оптерећењем:

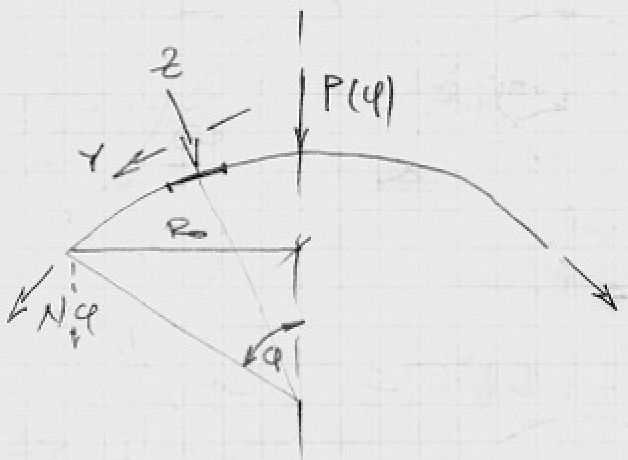
$N_\varphi, N_\theta \neq 0 \quad N_{\varphi\theta} \equiv 0 \quad$ зa $\quad X \equiv 0 \quad Y, Z = f(\varphi)$

$$N_\varphi = -\frac{1}{R_2 \sin^2\varphi} \cdot \left[\int R_1 R_2 (Y \sin\varphi + Z \cdot \cos\varphi) \sin\varphi \cdot d\varphi + C\right]$$

$$N_\varphi = -\frac{1}{R_0 \sin\varphi} \left[\int R_1 R_0 (Y \sin\varphi + Z \cdot \cos\varphi) d\varphi + C\right]$$

$2) \Rightarrow N_\theta = 0$





$\Sigma V = 0: \quad N_\varphi \cdot \sin\varphi \, 2\pi R_0 + P(\varphi) = 0$

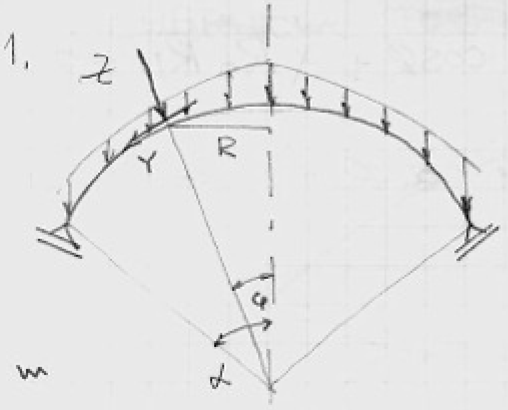
$$\boxed{N_\varphi = -\frac{P(\varphi)}{2\pi R_0 \sin\varphi}}$$

$$P(\varphi) = \int_0^\varphi (Y \sin\varphi + Z \cos\varphi) R_1 d\varphi \, 2\pi R_0 =$$

$$\boxed{= 2\pi \int_0^\varphi R_0 R_1 (Y \sin\varphi + Z \cos\varphi) d\varphi}$$

23.2.2006.

1.



γ-сп. т. материјала

$R_1 = R_2 = a$

$R_0 = a\sin\varphi$

$g = \gamma \cdot h \quad Z = g\cdot\cos\varphi$

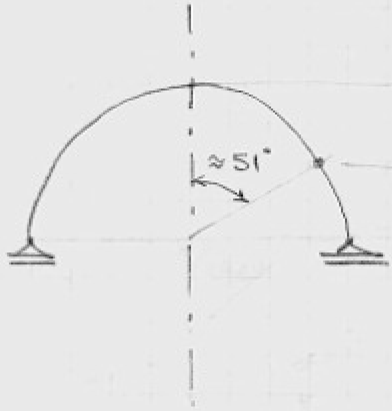
$Y = g\cdot\sin\varphi$

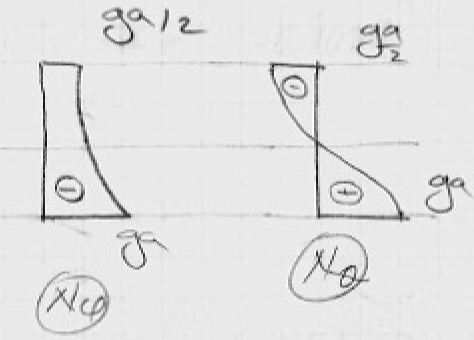
$$P(\varphi) = 2\pi\int_0^{\varphi} a^2\sin\varphi\,(g\sin^2\varphi + g\cos^2\varphi)\,d\varphi = 2\pi g a^2\int_0^{\varphi}\sin\varphi\,d\varphi$$

$$= -2\pi g a^2\cos\varphi\Big|_0^{\varphi} = -2\pi g a^2(\cos\varphi - 1)$$

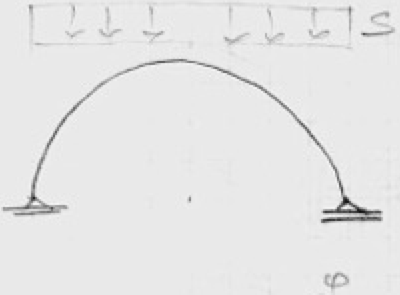
$$N_\varphi = \frac{2\pi g a^2(\cos\varphi - 1)}{2\pi a\sin^2\varphi} = -\frac{ga}{1+\cos\varphi}$$

$$\frac{N_\varphi}{a} + \frac{N_\theta}{a} + g\cos\varphi = 0 \qquad N_\theta = -ga\cos\varphi - N_\varphi = -ga\left(\cos\varphi - \frac{1}{1+\cos\varphi}\right)$$





2.



снијег $\quad s\,dx = s'\cdot ds \quad s' = s\cdot\frac{dx}{ds} = s\cdot\cos\varphi$

$Y = s\cdot\sin\varphi\cdot\cos\varphi$

$Z = s\cos^2\varphi$

$$P(\varphi) = 2\pi\cdot\int_0^{\varphi} a^2\sin\varphi\,(s\cdot\sin^2\varphi\cos\varphi + s\cos^3\varphi)\,d\varphi =$$

$$= 2\pi a^2 s\int_0^{\varphi}\sin\varphi\cos\varphi\,d\varphi = \pi a^2 s\int_0^{\varphi}\sin 2\varphi\,d\varphi =$$

$$= \pi a^2 s\int_0^{\varphi}\sin 2\varphi\,d\varphi = -\frac{\pi}{2}a^2 s\cos 2\varphi\Big|_0^{\varphi} = -\frac{\pi}{2}a^2 s(\cos 2\varphi$$

$$= \pi a^2 s\sin^2\varphi \qquad N_\varphi = -\frac{\pi a^2 s\cdot\sin^2\varphi}{2\pi a\sin^2\varphi} = -\frac{as}{2} \qquad N_\theta = -\frac{as}{2}\cos$$

2.3.2006.

$$Z=\gamma(h+a-a\cdot\cos\varphi)$$

$$P(\varphi)=2\pi\int_0^{\varphi}R_0R_1(Y\sin\varphi+Z\cos\varphi)\,d\varphi$$

$R_1=a \qquad R_0=a\sin\varphi$

$$P(\varphi)=2\pi a^2\gamma\left[(h+a)\int_0^{\varphi}\sin\varphi\cos\varphi\,d\varphi-a\int_0^{\varphi}\sin\varphi\cos^2\varphi\,d\varphi\right]$$

$Y=0$

$Z=f(\varphi)$

$Z=\gamma h_1$

$h_1=h+a-a\cos\varphi$

$$\int_0^{\varphi}\sin\varphi\cos^2\varphi\,d\varphi=-\cos^3\varphi\Big|_0^{\varphi}-2\cdot\int_0^{\varphi}\cos^2\varphi\sin\varphi\,d\varphi$$

$$u=\cos^2\varphi \quad du=-2\cdot\cos\varphi\sin\varphi\,d\varphi \;\Big|\; \begin{array}{l} dv=\sin\varphi\,d\varphi \\ v=-\cos\varphi \end{array}$$

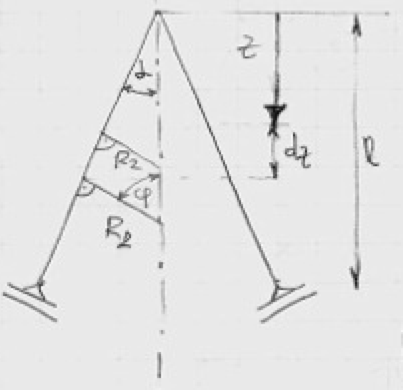
$$3\int_0^{\varphi}\sin\varphi\cos^2\varphi\,d\varphi=-(\cos^2\varphi-1)$$

$$\int_0^{\varphi}\sin\varphi\cos^2\varphi\,d\varphi=\frac{1}{3}(1-\cos^2\varphi)$$

$$P(\varphi)=2\pi a^2\gamma\left[\frac{h+a}{2}(1-\cos^2\varphi)-\frac{a}{3}(1-\cos^3\varphi)\right]$$

$$N(\varphi)=-\frac{P(\varphi)}{2\pi R_0\sin\varphi}$$

конусна љуска:

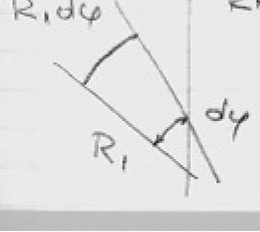


$$\varphi=\frac{\pi}{2}-\alpha=\text{const}$$

$$R_1\to\infty \qquad R_2=\frac{R_0}{\cos\alpha}$$

$$P(\varphi)=2\pi\int_0^{\varphi}R_0(Y\cdot\sin\varphi+Z\cdot\cos\varphi)R_1\,d\varphi$$

$$P(z)=2\pi\int_0^{z}z\,\mathrm{tg}\alpha(Y\cos\alpha+Z\sin\alpha)\frac{dz}{\cos\alpha}=$$



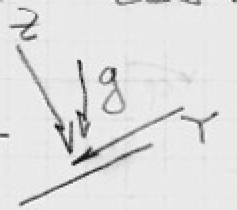
$$R_1d\varphi=\frac{dz}{\cos\alpha}$$

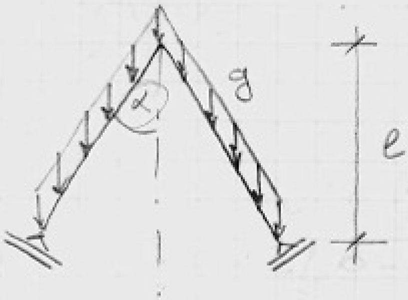
$$P(z)=2\pi\frac{\mathrm{tg}\alpha}{\cos\alpha}\int_0^{z}(Y\cos\alpha+Z\sin\alpha)\,z\,dz$$

$$N_\varphi = -\frac{P(z)}{2\pi R_0 \sin\varphi} = -\frac{P(z)}{2\pi z\, tg\alpha \cos\alpha} = -\frac{P(z)}{2\pi z \sin\alpha}$$

$$\frac{N_\varphi}{R_1} + \frac{N_\theta}{R_2} + Z = 0 \qquad \boxed{N_\theta = -Z R_2}$$

kott.

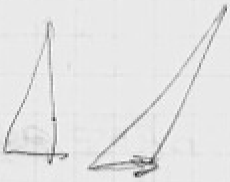




$g = \gamma \cdot h$

$Y = g \cdot \cos\alpha$

$Z = g \cdot \sin\alpha$

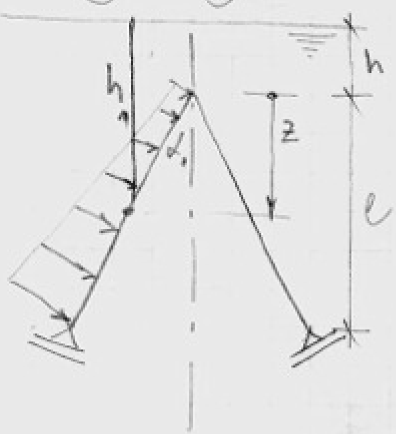


$$P(z) = 2\pi \cdot \frac{tg\alpha}{\cos\alpha} \int_0^z (g\cdot\cos^2\alpha + g\cdot\sin^2\alpha)\cdot z \cdot dz =$$

$$= 2\pi \frac{tg\alpha}{\cos\alpha}\, g \int_0^z z\,dz = 2\pi g \frac{tg\alpha}{\cos\alpha}\frac{z^2}{2} = \pi g z^2 \frac{tg\alpha}{\cos\alpha}$$

$$N_\varphi = \frac{\cancel{\pi} g z^{\cancel{2}} \frac{\cancel{tg\alpha}}{\cos\alpha}}{2\cancel{\pi}\cancel{z}\,\cancel{tg\alpha}\cos\alpha} = -\frac{gz}{2\cos^2\alpha} \qquad N_\theta = -R_2 Z = -\frac{z\cdot tg\alpha}{\cos\alpha} g \sin\alpha$$

$$= -gz\cdot tg^2\alpha$$



$Z = \gamma \cdot h_1 = \gamma\cdot(h+z)$

$Y = 0$

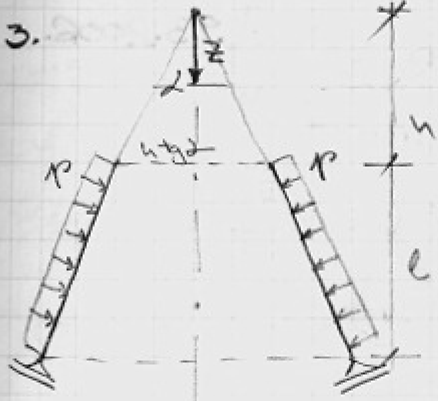
$$P(z) = 2\pi \frac{tg\alpha}{\cos\alpha}\int_0^z \gamma(h+z)\sin\alpha\, z\,dz =$$

$$= 2\pi\gamma\, tg^2\alpha \cdot z^2\left(\frac{h}{2} + \frac{z}{3}\right)$$

$$N_\varphi = \frac{2\pi\gamma\, tg^2\alpha\, z^2\left(\frac{h}{2}+\frac{z}{3}\right)}{2\pi z\, tg\alpha \cos\alpha}$$

$$N_\varphi = -\frac{\gamma z\cdot tg\alpha}{\cos\alpha}\left(\frac{h}{2} + \frac{z}{3}\right) \qquad N_\theta = -(h+z)\cdot\gamma\cdot z\frac{tg\alpha}{\cos\alpha}$$

3.



$Y = 0$

$Z = p$

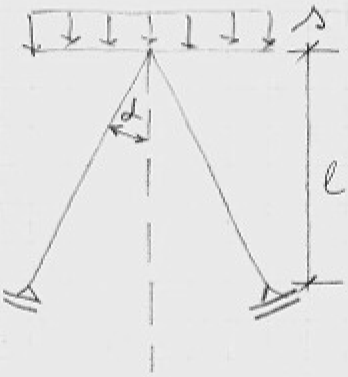
$R_0 \neq z \cdot tg\alpha$

$R_0 = h \cdot tg\alpha + z\, tg\alpha$

$$P(z) = 2\pi \frac{tg\alpha}{\cos\alpha} \int_h^z p \sin\alpha\, z\, dz =$$

$$= 2\pi\, tg^2\alpha \cdot p \cdot \frac{z^2}{2} \Big|_h^z = \pi p\, tg^2\alpha\, (z^2 - h^2)$$

$$N_\varphi = -\frac{\pi p\, tg^2\alpha\, (z^2 - h^2)}{2\pi \cdot z \cdot tg\alpha \cos\alpha} = -\frac{p}{2} \frac{tg\alpha}{\cos\alpha} \cdot \frac{z^2 - h^2}{z}$$



$Z = s \cdot \sin^2\alpha$

$Y = s \cdot \sin\alpha \cos\alpha$

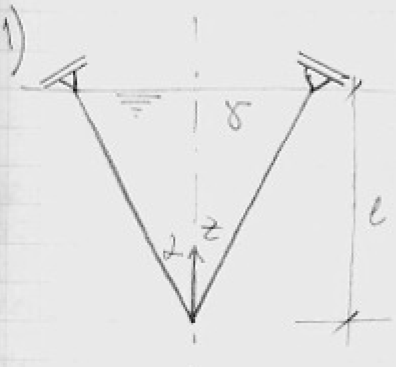
$$P(z) = 2\pi \frac{tg\alpha}{\cos\alpha} \int_0^z (s \sin\alpha \cos^2\alpha + s \cdot \sin^3\alpha)\, z \cdot dz$$

$$= 2\pi s \cdot tg^2\alpha \int_0^z z\, dz = s\pi\, tg^2\alpha \cdot z^2$$

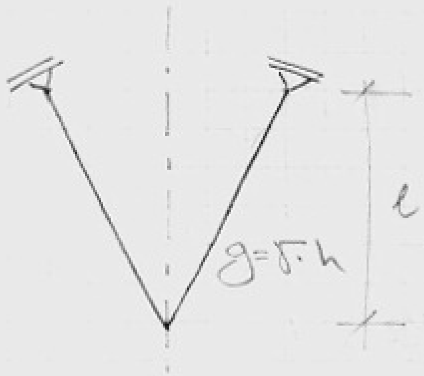
$$P(z) = R_0^2 \pi s = \pi s\, tg^2\alpha \cdot z^2$$

$$N_\varphi = -\frac{\pi s \cdot tg^2\alpha\, z^2}{2\pi z\, tg\alpha \cos\alpha} = -\frac{s}{2} z \frac{tg\alpha}{\cos\alpha} \qquad N_\theta = -R_2 Z$$

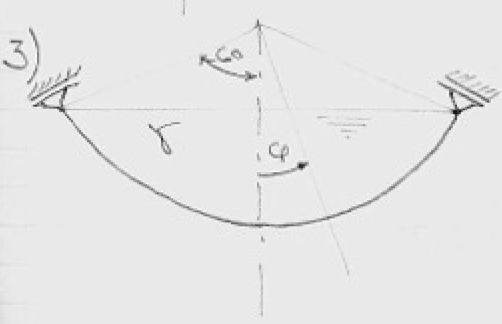
1)



2)



3)



9.3.2006.

Ротациона љуска оптерећена произвољним оптерећењем

$$X = \sum_1^\infty X_n \sin n\theta + \sum_0^\infty \bar{X}_n \cdot \cos n\theta$$
$$Y = \sum_0^\infty Y_n \cos n\theta + \sum_1^\infty \bar{Y}_n \sin n\theta$$
$$Z = \sum_0^\infty Z_n \cos n\theta + \sum_1^\infty \bar{Z}_n \sin n\theta$$

$\theta = 0$

$$x = X_n \sin n\theta \qquad N_\varphi = N_{\varphi n} \cos n\theta$$
$$Y = Y_n \cdot \cos n\theta \qquad N_\theta = N_{\theta n} \cos n\theta$$
$$Z = Z_n \cos n\theta \qquad N_{\varphi\theta} = N_{\theta n} \cdot \sin n\theta$$

$$\frac{d}{d\varphi}(R_0 N_{\varphi\theta n}) - n \cdot R_1 N_{\theta n} + R_1 N_{\varphi\theta n} \cos\varphi + X_n \cdot R_0 \cdot R_1 = 0$$

$$\frac{d}{d\varphi} \cdot (R_0 N_{\varphi n}) + n R_1 \cdot N_{\varphi\theta n} - R_1 N_{\theta n} \cos\varphi + Y_n \cdot R_0 R_1 = 0$$

$$\frac{N_{\varphi n}}{R_1} + \frac{N_{\theta n}}{R_2} + Z_n = 0$$

$$R_1 = R_2 = a \qquad R_0 = a \cdot \sin\varphi$$

(1) $$\frac{dN_{\varphi\theta n}}{d\varphi} + 2 \cdot \operatorname{ctg}\varphi\, N_{\varphi\theta n} + \frac{n}{\sin\varphi} N_{\varphi n} + a\left(X_n + \frac{n}{\sin\varphi} Z_n\right) = 0$$

(2) $$\frac{dN_{\varphi n}}{d\varphi} + 2 \cdot \operatorname{ctg}\varphi\, N_{\varphi n} + \frac{n}{\sin\varphi} N_{\varphi\theta n} + a\left(Y_n + \operatorname{ctg}\varphi\, Z_n\right) = 0$$

$$u_1 = N_{\varphi n} + N_{\varphi\theta n}$$
$$u_2 = N_{\varphi n} + N_{\varphi\theta n}$$

(1) + (2) $$\frac{du_1}{d\varphi} + \left(2 \cdot \operatorname{ctg}\varphi + \frac{n}{\sin\varphi}\right) u_1 + a\left(X_n + Y_n + \frac{n + \cos\varphi}{\sin\varphi} Z_n\right)$$

(1) − (2) $$\frac{du_2}{d\varphi} + \left(2 \cdot \operatorname{ctg}\varphi - \frac{n}{\sin\varphi}\right) u_2 + a \cdot \left(Y_n - X_n - \frac{n - \cos\varphi}{\sin\varphi} Z_n\right)$$

Линеарне диференц. једначине:

$$\frac{du_k}{d\varphi} + p_k(\varphi) \cdot u_k + g_k(\varphi) = 0 \quad ; \quad k = 1, 2 \ldots$$

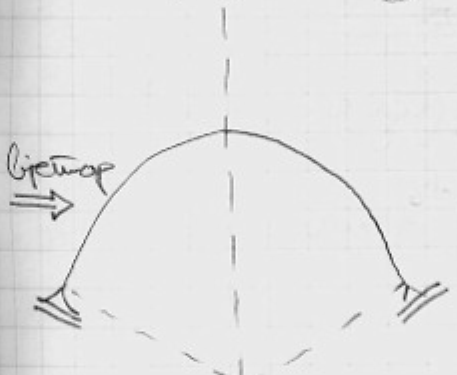
$$u_k = \left[C_k - \int g_k \cdot e^{\int p_k \cdot d\varphi} d\varphi\right] \cdot e^{-\int p_k d\varphi}$$

$$u_1 = \frac{\operatorname{ctg}\frac{n\varphi}{2}}{\sin^2\varphi}\left[c_1 - a\int\left(X_n + Y_n + \frac{n+\cos\varphi}{\sin\varphi}\cdot Z_n\right)\sin^2\varphi\,\operatorname{tg}\frac{n\varphi}{2}\,d\varphi\right]$$

$$u_2 = \frac{\operatorname{tg}\frac{n\varphi}{2}}{\sin^2\varphi}\left[c_2 - a\int\left(-X_n + Y_n - \frac{n-\cos\varphi}{\sin\varphi}Z_n\right)\sin^2\varphi\right.$$

$$\left.\operatorname{ctg}\frac{n\varphi}{2}\,d\varphi\right]$$

1. сферна љуска оптерећена вјетром



$X = Y = 0$

$Z = \sum_0^\infty p_n \sin\varphi\, k_n \cos n\theta$

$n=1 \quad Z_n = p_n \cdot k_n \cdot \sin\varphi$

$Z_n = p \cdot \sin\varphi$

$$u_1 = \frac{1+\cos\varphi}{\sin^3\varphi}\left[c_1 + pa\left(\cos\varphi - \frac{1}{3}\cdot\cos^3\varphi\right)\right]$$

$$u_2 = \frac{1-\cos\varphi}{\sin^3\varphi}\left[c_2 - pa\left(\cos\varphi - \frac{1}{3}\cos^3\varphi\right)\right]$$

$$N_\varphi = \frac{\cos\theta}{\sin^3\varphi}\left[\frac{c_1+c_2}{2} + \frac{c_1-c_2}{2}\cos\varphi + pa\left(\cos^2\varphi - \frac{1}{3}\cos^4\varphi\right)\right]$$

$$N_{\varphi\theta} = \frac{\sin\theta}{\sin^3\varphi}\left[\frac{c_1-c_2}{2} + \frac{c_1+c_2}{2}\cdot\cos\varphi + pa\left(\cos\varphi - \frac{1}{3}\cos^3\varphi\right)\right]$$

$\varphi = 0 \qquad \sin^3\varphi = 0$ — тражимо изводе до 2. реда $\sin^3\varphi$ који нам смета

$3\sin^2\varphi\cos\varphi = 0$

$(\sin^3\varphi)'' = 0$

$\varphi = 0$:

$$\frac{c_1+c_2}{2} + \frac{c_1-c_2}{2} + p\cdot a\cdot\frac{2}{3} = 0$$

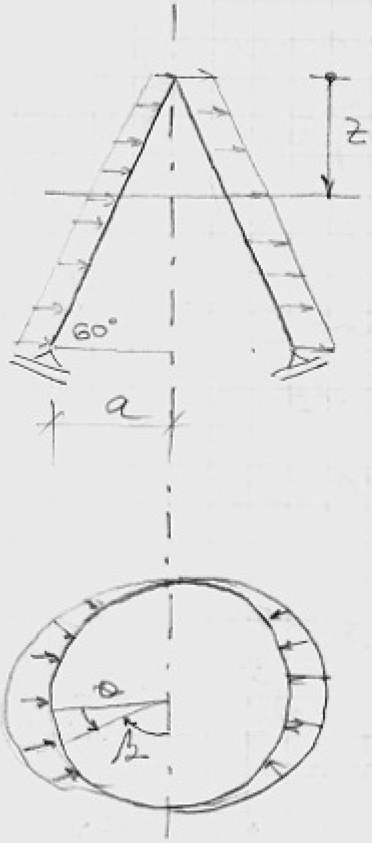
$$\frac{c_1-c_2}{2} + \frac{c_1+c_2}{2} + p\cdot a\cdot\frac{2}{3} = 0 \qquad c_1 = -\frac{2}{3}pa$$

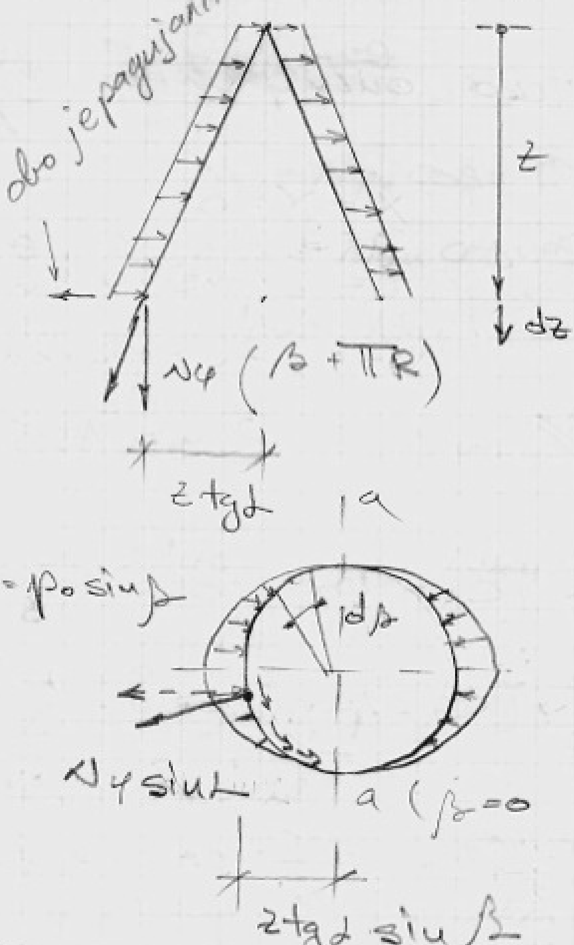
$$-\frac{c_1-c_2}{2}\sin\varphi + p\cdot a\left(-2\cdot\cos\varphi\sin\varphi + \frac{4}{3}\cos^3\varphi\sin\varphi\right)$$

$$-\frac{C_1-C_2}{2}\cos\varphi + p\cdot a\left(-2\cdot\cos 2\varphi + \frac{4}{3}\cos^4\varphi - 4\cos^2\varphi\cdot\sin^2\varphi\right)$$

$$\varphi=0:\quad -\frac{C_1-C_2}{2} + p\cdot a\cdot\left(-\frac{2}{3}\right)=0 \qquad C_2=\frac{2}{3}p\cdot a$$

ово је радијална компонента! НЕ хоризонтална





$p_н = p_0 \sin\beta$

$N\varphi = N\varphi_0\cdot\sin\beta$

$N\varphi\theta = N\varphi\theta_0 \cos\beta_0$

$f(z)$

$$H=\int_F p_н\cdot dF = \int_0^z\int_0^{2\pi} p_0\sin^2\beta\cdot z\cdot tg\alpha\cdot\frac{dz}{\cos\alpha}\,d\beta$$

$$H=\frac{p_0\pi}{2}\cdot z^2\cdot\frac{tg\alpha}{\cos\alpha}$$

$$\Sigma H=0:\quad H+\int_0^{2\pi}N\varphi\theta\cdot\cos\beta\cdot z\, tg\alpha\cdot d\beta - \int_0^{2\pi}N\varphi\cdot\sin\alpha\sin\beta\cdot z\, tg\alpha\cdot d\beta=0 :$$

$z = \frac{2}{3}$ (за домаћи)

$\Sigma M_{a-a} = 0$ (моменат даје само сила N_φ а $N_{\varphi\theta}$ не даје! Из овог услова се одреди N_φ)

$$H \cdot \frac{z}{3} - \int_0^{2\pi} \underbrace{N_\varphi}_{= N_{\varphi 0}\cdot \sin\beta} \cdot \cos\alpha \cdot z \cdot tg\alpha \sin\beta \cdot z \cdot tg\alpha \, d\beta = 0$$

$$\boxed{N_{\varphi 0} = -\frac{p_0 \cdot z}{3 \cdot \sin\alpha}} \qquad N_\theta = -R \cdot z$$

Деформација средње површи ротационе љуске

- ротационо симетрично оптерећене

N_φ, N_θ

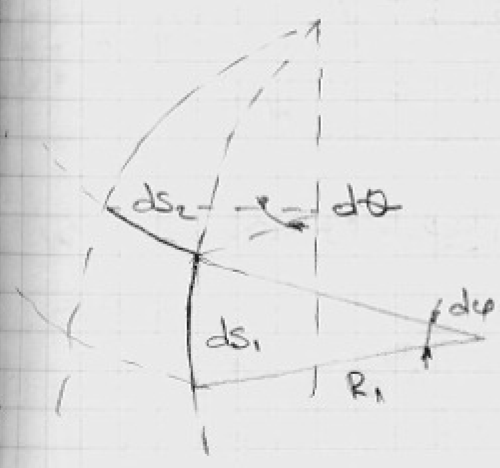
u, v, w

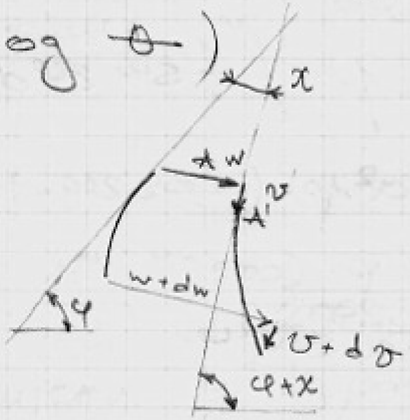
↓

u - у правцу тангенте на круг (у правцу θ)

v - у правцу тангенте на мериц. криву

$u \equiv 0$ (код ротационе симетрије ништа не зависи од θ)





$$ds_1 = R_1 d\varphi \qquad \varepsilon_\varphi = \frac{1}{Eh}(N_\varphi - \nu \cdot N_\theta)$$

$$ds_2 = R_0 \cdot d\theta \qquad \varepsilon_\theta = \frac{1}{Eh}\cdot(N_\theta - \nu \cdot N_\varphi)$$

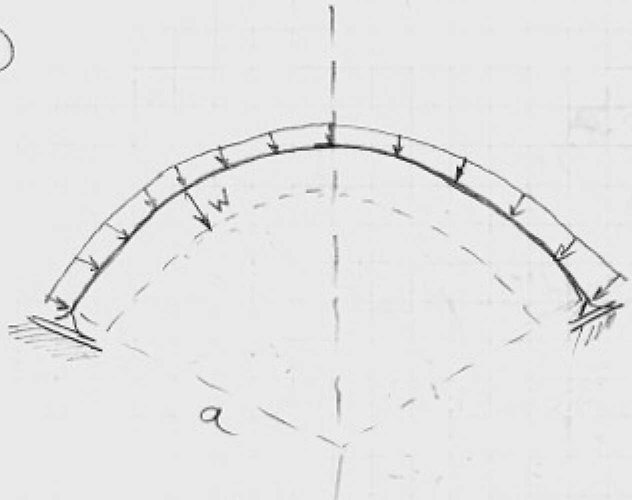
$$\varepsilon_\varphi = \frac{1}{R_1}\left(\frac{dv}{d\varphi} - w\right)$$

$$\varepsilon_\theta = \frac{\Delta R_0}{R_0} = \frac{1}{R_2}(v \cdot ctg\varphi - w)$$

$$v = \left[\int \frac{R_1 \cdot \varepsilon_\varphi - R_2 \cdot \varepsilon_\theta}{\sin\varphi} d\varphi + C \right] \sin\varphi$$

$$\chi = \frac{1}{R_1}\left(v + \frac{dw}{d\varphi}\right)$$ угао промјене нагиба тангенте

(1)



$w \neq 0 \quad \chi \equiv 0$

$v \equiv 0$

$Z = p$

$Y = 0$

$R_0 = a \sin\varphi$

$R_1 = a$

$$P(\varphi) = -\frac{1}{2}\pi a^2 \cdot p\,(\cos 2\varphi - 1) \qquad N_\varphi = -\frac{P(\varphi)}{2\pi R_0 \sin\varphi}$$

$$\frac{N_\varphi}{a} + \frac{N_\theta}{a} + Z = 0$$

$$P(\varphi) = 2\pi \int_0^\varphi R_0 R_1 \cdot Z \cos\varphi \, d\varphi = 2\pi \int_0^\varphi a^2 \sin\varphi \, p \cdot \cos\varphi \, d\varphi$$

$$P(\varphi) = 2\pi a^2 \cdot p \cdot \int_0^\varphi \frac{1}{2} \sin 2\varphi \, d\varphi = -\frac{1}{2}\pi a^2 p \cos 2\varphi \Big|_0^\varphi$$

$$P(\varphi) = -\frac{1}{2}\pi a^2 p \cdot (\cos 2\varphi - 1)$$

$$N_\varphi = \frac{\frac{1}{2}\pi a^2 p \cdot (\cos 2\varphi - 1)}{2\pi a \sin^2\varphi}, \qquad 2 \cdot \sin^2\varphi = 1 - \cos 2\varphi$$

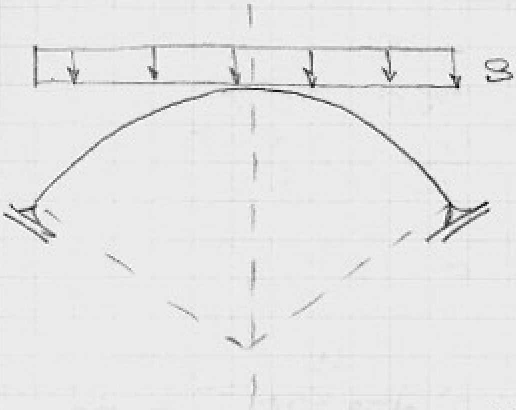
$$\boxed{N_\varphi = -\frac{pa}{2}} \qquad N_\theta = -Z \cdot a - N_\varphi \qquad \boxed{N_\theta = -\frac{pa}{2}}$$

$$\varepsilon_\varphi = \frac{1}{Eh}(N_\varphi - \nu \cdot N_\theta) = \frac{pa \cdot (\nu - 1)}{2Eh} \ldots$$

$$\varepsilon_\theta = \frac{1}{Eh}(N_\theta - \nu \cdot N_\varphi) = \frac{pa \cdot (\nu - 1)}{2Eh} \qquad \Delta R_0 = \varepsilon_\theta \cdot R_0$$

$$\Delta R_0 = \frac{pa \cdot (\nu - 1)}{2Eh} \cdot a \cdot \sin\varphi$$

2.



$\chi = ?$

$\Delta R_0 = ?$

$\chi = \frac{1}{a}\left(v + \frac{dw}{d\varphi}\right)$

$N_\varphi = -pa/2$; $N_\theta = -\frac{p\cdot a}{2}\cos 2\varphi$

$$\varepsilon_\varphi = \frac{1}{Eh}\left(-\frac{p\cdot a}{2} + \nu\cdot\frac{p\cdot a}{2}\cdot\cos 2\varphi\right) = \frac{p\cdot a}{2\cdot Eh}(\nu\cdot\cos 2\varphi - 1)$$

$$\varepsilon_\theta = \frac{1}{Eh}\cdot\left(-\frac{p\cdot a}{2}\cos 2\varphi + \nu\cdot\frac{p\cdot a}{2}\right) = \frac{p\cdot a}{2Eh}(\nu - \cos 2\varphi)$$

$$\Delta R_0 = \varepsilon_\theta\cdot R_0 = \frac{pa}{2Eh}\cdot(\nu - \cos 2\varphi)\cdot a\cdot\sin\varphi$$

$$\Delta R_0 = \frac{pa^2}{Eh}\sin\varphi\left(\frac{1+\nu}{2} - \cos^2\varphi\right)$$

$$v = \sin\varphi\int\frac{a\cdot\left[\frac{pa}{2Eh}(\nu\cdot\cos 2\varphi - 1) - \frac{pa}{2Eh}(\nu - \cos^2\varphi)\right]}{\sin\varphi}\,d\varphi$$

$$v = \frac{pa^2}{2Eh}\sin\varphi\int\frac{\nu\cdot\cos 2\varphi - 1 - \nu + \cos 2\varphi}{\sin\varphi}\,d\varphi =$$

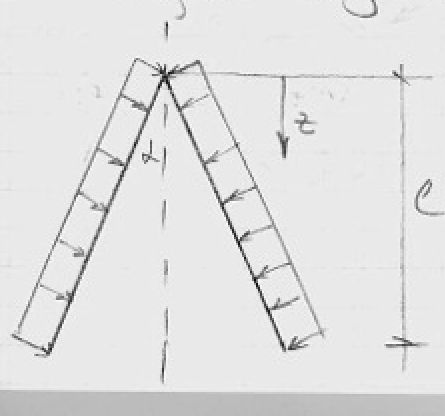
$$\boxed{v = \frac{pa^2\cdot(1+\nu)}{Eh}\sin\varphi\cos\varphi}$$

$$w = v\cdot ctg\,\varphi - \varepsilon_\theta\cdot a = \frac{pa^2}{2Eh}(1 + 2\cos 2\varphi + \nu\cdot\cos 2\varphi)$$

Нађе се $\frac{dw}{d\varphi}$ и замијени у израз за χ

$$\chi = -\frac{pa}{2Eh}\cdot(3+\nu)\cdot\sin 2\varphi$$

3. конусна љуска:



$Z = p$

$Y = 0$

$$\varepsilon_\varphi = \frac{dv}{dz}\cos\alpha$$

$$\chi = \frac{dw}{dz}\cdot\cos\alpha$$

$$w = \frac{1}{\cos\alpha}\left(v\cdot\sin\alpha - R_0\,\varepsilon_\theta\right)$$

$$P(z) = \frac{2\pi\,\mathrm{tg}\,\alpha}{\cos\alpha}\int_0^z(\gamma\cos\alpha + z\cdot\sin\alpha)\,z\cdot dz = p\pi z^2\,\mathrm{tg}^2\alpha$$

$$N_\varphi = -\frac{p\pi z^2\,\mathrm{tg}^2\alpha}{2\pi z\,\mathrm{tg}\,\alpha\cos\alpha} = -\frac{pz\cdot\mathrm{tg}\,\alpha}{2\cos\alpha}$$

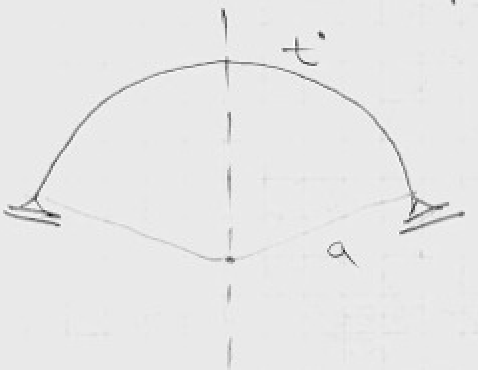
$$N_\theta = -Z\cdot R_2 = -p_0\cdot\frac{R_0}{\cos\alpha} = -\frac{pz\cdot\mathrm{tg}\,\alpha}{\cos\alpha}$$

$$\varepsilon_\varphi = \frac{1}{Eh}\left(-\frac{pz\,\mathrm{tg}\,\alpha}{2\cdot\cos\alpha} + \nu\cdot\frac{p\cdot z\cdot\mathrm{tg}\,\alpha}{\cos\alpha}\right) = \frac{p\cdot z\,\mathrm{tg}\,\alpha}{2Eh\cdot\cos\alpha}(2\nu-1)$$

$$\varepsilon_\theta = \frac{1}{Eh}\left(-\frac{pz\,\mathrm{tg}\,\alpha}{\cos\alpha} + \nu\cdot\frac{pz\,\mathrm{tg}\,\alpha}{2\cdot\cos\alpha}\right) = \frac{pz\,\mathrm{tg}\,\alpha}{2Eh\cos\alpha}(\nu-2)$$

$$\Delta R_0 = R_0\cdot\varepsilon_\theta = z\,\mathrm{tg}\,\alpha\cdot\frac{pz\,\mathrm{tg}\,\alpha}{2Eh\cdot\cos\alpha}(\nu-2)$$ $\chi = ?$ (gom...

ПРИМЈЕР: Мембранско стање напрезања



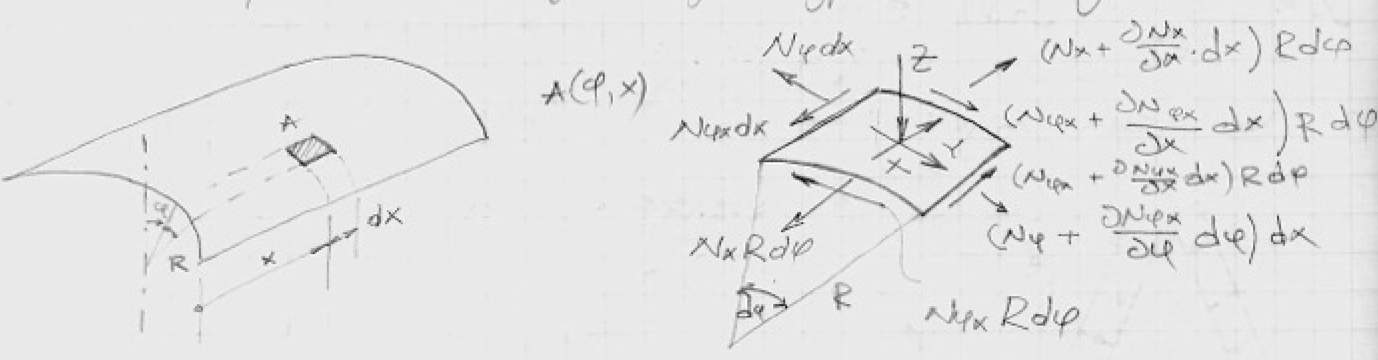
$$\Delta R_0 = R_0\cdot\alpha_t\cdot t$$

$$\chi = 0$$

$$\varepsilon_\varphi = \varepsilon_\theta = \alpha_t\cdot t$$

23. 3. 2006.

Мембранска теорија цилиндричних љуски:



$$\Sigma X = 0 \quad \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{R\partial\varphi} + X = 0$$

$$\Sigma Y = 0 \quad \frac{\partial N_\varphi}{R\partial\varphi} + \frac{\partial N_{\varphi x}}{\partial x} + Y = 0$$

$$\Sigma Z = 0 \quad \frac{N_\varphi}{R} + Z = 0 \Rightarrow N_\varphi = -R\cdot Z$$

$$N_{\varphi x} = -\int \left(Y + \frac{\partial N_\varphi}{R\partial\varphi}\right) dx + C_1(\varphi)$$

$$N_x = -\int \left(X + \frac{\partial N_{\varphi x}}{R\partial\varphi}\right) dx + C_2(\varphi)$$

$X \equiv 0$

$$Y, Z = f(\varphi) \quad N_{\varphi x} = -\left(Y + \frac{1}{R}\cdot\frac{dN_\varphi}{d\varphi}\right)\cdot x + C_1(x)$$

$$N_x = \frac{1}{R}\cdot\frac{d}{d\varphi}\left(Y + \frac{1}{R}\frac{dN_\varphi}{d\varphi}\right)\frac{x^2}{2} - \frac{1}{R}\frac{dC_1(x)}{d\varphi}x + C_2(x)$$

$$X = \sum_{n=0}^{\infty} \overbrace{X_n}^{f(x)} \cos n\varphi \qquad Y = \sum_{n=1}^{\infty} Y_n \sin n\varphi \qquad Z = \sum_{n=0}^{\infty} Z_n \cos n\varphi$$

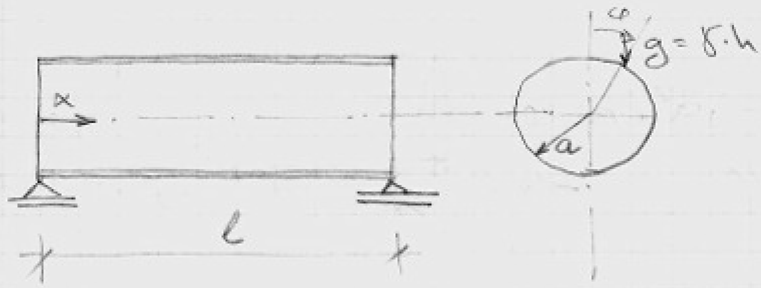
$$N_\varphi = -a\cdot Z\cdot\cos n\varphi \qquad C_1(\varphi) = A_1\cdot\sin n\varphi$$

$$C_2(\varphi) = A_2 \cos n\varphi$$

$$N_{\varphi x} = -\left[(Y_n + n\cdot Z_n)\,x - A_1\right]\cdot\sin n\varphi$$

$$N_x = \left\{\frac{n}{a}\left[(Y_n + nZ_n)\frac{x^2}{2} - A_1 x\right] + A_2\right\}\cos n\varphi$$

(1)



$x = 0 \; : \; N_x = 0$

$x = l \quad N_x = 0$

$A_2 = 0$

$A_1 = g\cdot l$

$h, E, \nu \qquad n = 1$

$X \equiv 0 \qquad N_\varphi = -a\cdot g\cdot\cos\varphi$

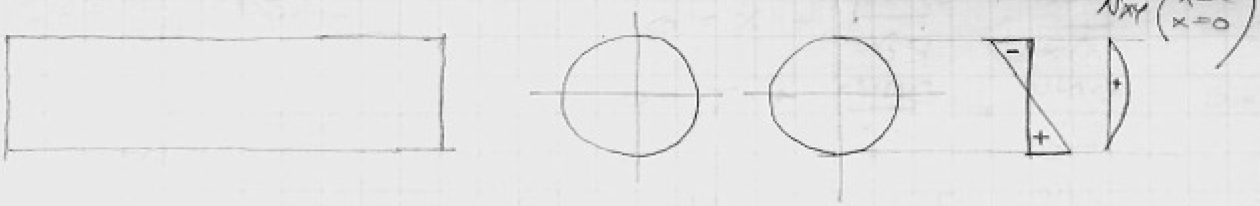
$Y = g\cdot\sin\varphi \qquad N_{\varphi x} = -(2gx - A_1)\sin\varphi = g\cdot(l - 2x)\cdot\sin\varphi$

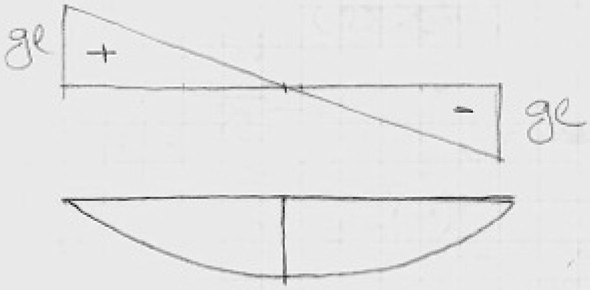
$Z = g\cdot\cos\varphi \qquad N_x = \left(\frac{g}{a}\frac{x^2}{\cancel{2}} - \frac{A_1}{a}x + A_2\right)\cos\varphi = -\frac{g}{a}\cdot x(l - x)\cos\varphi$

$$N_\varphi = -a\cdot Z_n\cdot\cos n\varphi$$

$$N_{\varphi x} = -\left[(Y_n + nZ_n)\cdot x - A_1\right]\cdot\sin n\varphi$$

$$N_x = \left\{\frac{n}{a}\left[(Y_n + n\cdot Z_n)\frac{x^2}{2} - A_1 x\right] + A_2\right\}\cos n\varphi$$



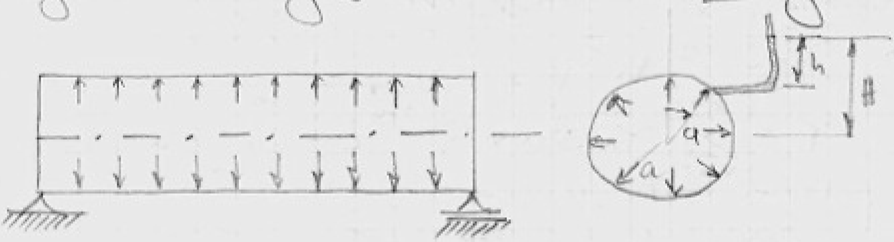


$N_{\varphi x}(\varphi=\frac{\pi}{2})$

$N_x(\varphi=\pi)$

7) ПРИМЈЕР:

Љуска испуњена течношћу



$x=0 \quad N_x=0$

$x=l \quad N_x=0$

$Z=-\gamma h=-\gamma\cdot(H-a\cdot\cos\varphi)=-\gamma H-a\cdot\gamma\cdot\cos\varphi$

$N_{\varphi x}=N_x\equiv 0$

$N_{\varphi}=-Z\cdot a=\gamma H\cdot a \qquad A_2=0 \qquad A_1=\frac{\gamma a\cdot l}{2}$

$N_{\varphi x}=-(\gamma a x-\frac{\gamma a l}{2})\sin\varphi$

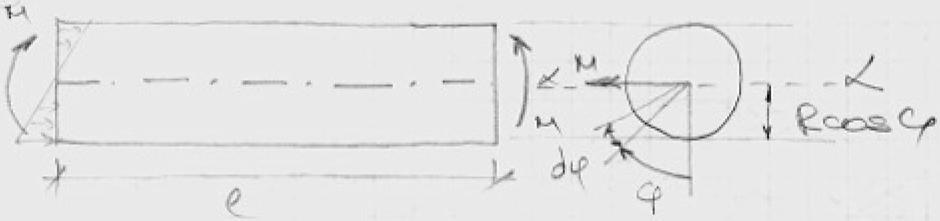
$N_x=\frac{\gamma}{2}x(x-l)\cos\varphi$

$Z=\gamma a\cdot\cos\varphi \qquad n=1$

$X\equiv Y\equiv 0 \qquad N_{\varphi}=-Z\cdot a=-\gamma a^2\cos\varphi$

$N_{\varphi x}=-(\gamma a\cdot x-A_1)\cdot\sin\varphi \qquad N_x=\left[\frac{1}{a}\left(\gamma a\cdot\frac{x^2}{2}-A_1 x\right)+A_2\right]\cos\varphi$

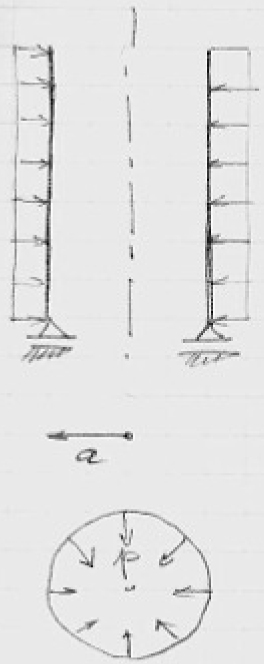
3.



$X=Y=Z\equiv 0 \qquad N_{\varphi x}=-A_1\sin\varphi$

$N_{\varphi}\equiv 0 \qquad N_x=\left(-\frac{A_1}{a}\cdot x+A_2\right)\cos\varphi$

(4)



$Z = - p$

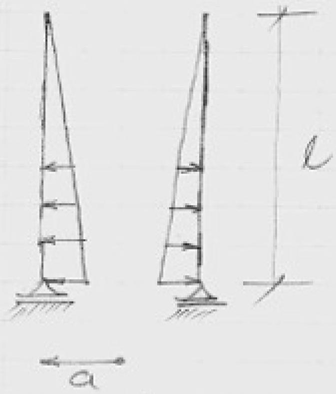
$X = Y = 0$

$N_\varphi = - p \cdot a$

$\varepsilon_\varphi = - \dfrac{w}{a} \qquad \varepsilon_\varphi = \dfrac{1}{h}(N_\varphi - \nu \cdot N_x) = \dfrac{N_\varphi}{Eh} = - \dfrac{pa}{Eh}$

$w = \dfrac{pa^2}{Eh} = \Delta R_0 \qquad \chi = 0$

(5).



$Z = - \gamma \cdot (l - x) \qquad \Delta R_0 = - \dfrac{\gamma a^2}{Eh}(l - x) = w$

$N_\varphi = \gamma \cdot a (l - x) \qquad \chi = \dfrac{dw}{dx} = \gamma a^2 / Eh = \text{const}$

$\varepsilon_\varphi = \dfrac{\gamma a}{Eh}(l - x)$

Δομητικη :

6.